STEEL FORMING AND HEAT TREATING HANDBOOK

Antonio Augusto Gorni
São Vicente SP
Brazil

agorni@iron.com.br
www.gorni.eng.br

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FOREWORD

This is a compilation of some useful mathematical formulas, graphics and data in the area of forming, heat treatment and physical metallurgy of steels. The very first version arose in the early eighties, as a handwritten sheet with a few formulas. Afterwards it was converted to a digital format and eventually posted on-line, hoping that it could be also helpful worldwide. It must be noted that these formulas were compiled at random, generally in a need-to-know basis. So, this Handbook is in permanent construction and very far to be complete. Finally, the author thanks Seok-Jae Lee, Assistant Professor of the Chonbuk National University, Republic of Korea, for his contribution.

DISCLAIMER

The formulas and information compiled in this text are provided without warranty of any kind.

Use them at your own risk!

However, any help regarding the correction of eventual mistakes is appreciated.

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- **Austenite Formation Temperatures**

. Andrews

$$Ae_3 = 910 - 25\,Mn - 11\,Cr - 20\,Cu + 60\,Si + 60\,Mo + 40\,W + 100\,V + 700\,P$$

$$Ac_1 = 723 - 10.7\,Mn - 16.9\,Ni + 29.1\,Si + 16.9\,Cr + 290\,As + 6.38\,W$$

$$Ac_3 = 910 - 203\,\sqrt{C} - 15.2\,Ni + 44.7\,Si + 104\,V + 31.5\,Mo + 13.1\,W$$

Notation:

Ae_3: Lower Equilibrium Temperature Between Ferrite and Austenite [°C]
Ac_1: Upper Heating Temperature Between Ferrite and Austenite [°C]
Ac_3: Lower Heating Temperature Between Ferrite and Austenite [°C]
Alloy Content: [weight %]

Source: ANDREWS, K.W. *Empirical Formulae for the Calculation of Some Transformation Temperatures.* **Journal of the Iron and Steel Institute**, 203, Part 7, July 1965, 721-727.

. Brandis

$$Ac_1 = 739 - 22\,C - 7\,Mn + 2\,Si + 14\,Cr + 13\,Mo - 13\,Ni + 20\,V$$

$$Ac_3 = 902 - 255\,C - 11\,Mn + 19\,Si - 5\,Cr + 13\,Mo - 20\,Ni + 55\,V$$

Notation:

Ac_1: Lower Heating Temperature Between Ferrite and Austenite [°C]
Ac_3: Upper Heating Temperature Between Ferrite and Austenite [°C]
Alloy Content: [weight %]

Source: BRANDIS, H. *Rechnerische Bestimmung der Umwandlungstemperaturen von niedriglegierten Stählen.* **TEW – Technische Berichte**, Band 1, Heft 1, 1975, 8-10.

. Eldis

$$Ae_1 = 712 - 17.8\ Mn - 19.1\ Ni + 20.1\ Si + 11.9\ Cr + 9.8\ Mo$$

$$Ae_3 = 871 - 254.4\ \sqrt{C} - 14.2\ Ni + 51.7\ Si$$

Notation:

Ae_1: Lower Equilibrium Temperature Between Ferrite and Austenite [°C]
Ae_3: Upper Equilibrium Temperature Between Ferrite and Austenite [°C]
Alloy Content: [weight %]

Observations:

- Both formulas were proposed by ELDIS for low alloy steels with less than 0.6%C.

Source: BARRALIS, J. & MAEDER, G. *Métallurgie Tome I: Métallurgie Physique.* **Collection Scientifique ENSAM**, 1982, 270 p.

. Grange

$$Ae_1 = 1333 - 25\ Mn + 40\ Si + 42\ Cr - 26\ Ni$$

$$Ae_3 = 1570 - 323\ C - 25\ Mn + 80\ Si - 3\ Cr - 32\ Ni$$

Notation:

Ae_1: Lower Equilibrium Temperature Between Ferrite and Austenite [°F]
Ae_3: Upper Equilibrium Temperature Between Ferrite and Austenite [°F]

Alloy Content: [weight %]

Source: GRANGE, R.A. *Estimating Critical Ranges in Heat Treatment of Steels*. **Metal Progress**, 70:4, April 1961, 73-75.

. Hougardy

$$Ac_1 = 739 - 22\,C - 7\,Mn + 2\,Si + 14\,Cr + 13\,Mo - 13\,Ni$$

$$Ac_3 = 902 - 255\,C - 11\,Mn + 19\,Si - 5\,Cr + 13\,Mo - 20\,Ni + 55\,V$$

Notation:

Ac_1: Lower Temperature of the Ferrite-Austenite Field During Heating [°C]
Ac_3: Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
Alloy Content: [weight %]

Source: HOUGARDY, H.P. **Werkstoffkunde Stahl Band 1: Grundlagen**. Verlag Stahleisen GmbH, Düsseldorf, 1984, p. 229.

. Kasatkin

$$\begin{aligned} Ac_1 = {} & 723 - 7.08\,Mn + 37.7\,Si + 18.1\,Cr + 44.2\,Mo + 8.95\,Ni + 50.1\,V + 21.7\,Al + 3.18\,W + 297\,S - 830\,N - 11.5\,C\,Si - 14.0\,Mn\,Si - \\ & - 3.10\,Si\,Cr - 57.9\,C\,Mo - 15.5\,Mn\,Mo - 5.28\,C\,Ni - 6.0\,Mn\,Ni + 6.77\,Si\,Ni - 0.80\,Cr\,Ni - 27.4\,C\,V + 30.8\,Mo\,V - 0.84\,Cr^2 - \\ & - 3.46\,Mo^2 - 0.46\,Ni^2 - 28\,V^2 \end{aligned}$$

Observations:

- Multiple Correlation Coefficient r = 0.96
- Residual Mean-Square Deviation $\sqrt{d_o}$ = 10.8°C

$$Ac_3 = 912 - 370\,C - 27.4\,Mn + 27.3\,Si - 6.35\,Cr - 32.7\,Ni + 95.2\,V + 190\,Ti + 72.0\,Al + 64.5\,Nb + 5.57\,W + 332\,S + 276\,P + 485\,N - 900\,B + 16.2\,C\,Mn + 32.3\,C\,Si + 15.4\,C\,Cr + 48.0\,C\,Ni + 4.32\,Si\,Cr - 17.3\,Si\,Mo - 18.6\,Si\,Ni + 4.80\,Mn\,Ni + 40.5\,Mo\,V + 174\,C^2 + 2.46\,Mn^2 - 6.86\,Si^2 + 0.322\,Cr^2 + 9.90\,Mo^2 + 1.24\,Ni^2 - 60.2\,V^2$$

Observations:

- Multiple Correlation Coefficient r = 0.98
- Residual Mean-Square Deviation $\sqrt{d_o}$ = 14.5°C

$$\Delta T = 188 - 370\,C - 7.93\,Mn - 26.8\,Cr - 33.0\,Mo - 23.5\,Ni + 52.5\,V + 194\,Ti + 47.8\,Al + 87.4\,Nb + 3.82\,W + 266\,P + 53.0\,C\,Si + 20.7\,C\,Cr + 6.26\,Si\,Cr + 64.2\,C\,Mo + 55.2\,C\,Ni + 10.8\,Mn\,Ni + 1.33\,Cr^2 + 8.83\,Mo^2 + 1.91\,Ni^2 - 37.8\,V^2$$

Notation:

Ac_1: Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
Ac_3: Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
ΔT: Intercritical Temperature Range [°C]
Alloy Content: [weight %]

Observations:

- Multiple Correlation Coefficient r = 0.97
- Residual Mean-Square Deviation $\sqrt{d_o}$ = 16.8°C
- These equations (**Ac_1**, **Ac_3**, **ΔT**) are valid within these composition limits: C ≤ 0.83%, Mn ≤ 2.0%, Si ≤ 1.0%, Cr ≤ 2.0%, Mo ≤ 1.0%, Ni ≤ 3.0%, V ≤ 0.5%, W ≤ 1.0%, Ti ≤ 0.15%, Al ≤ 0.2%, Cu ≤ 1.0%, Nb ≤ 0.20%, P ≤ 0.040%, S ≤ 0.040%, N ≤ 0.025%, B ≤ 0.010%.

Source: KASATKIN, O.G. et alii. *Calculation Models for Determining the Critical Points of Steel.* **Metal Science and Heat Treatment**, 26:1-2, January-February 1984, 27-31.

. Kunitake & Katou

$$Ac_1 = 754.83 - 32.25\,C - 17.76\,Mn + 23.32\,Si + 17.3\,Cr + 4.51\,Mo + 15.62\,V$$

$$Ac_3 = 920.21 - 394.75\,C - 14.40\,Mn + 54.99\,Si + 5.77\,Cr + 24.49\,Mo + 83.37\,V$$

$$\delta = 0.0617 + 0.233\,C + 0.025\,Mn - 0.049\,Si + 0.027\,Cr - 0.050\,Mo - 0.061\,V$$

Notation:

Ac_1: Lower Temperature of the Ferrite-Austenite Field During Heating [°C]
Ac_3: Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
δ: Length Change Due to Transformation [%]
Alloy Content: [weight %]

Observations:

- Equation valid within the following alloy range: 0.25% ≤ **C** ≤ 0.45%; 0.7 ≤ **Mn** ≤ 1.2%; 0.6 ≤ **Si** ≤ 3.1%; 0.8 ≤ **Cr** ≤ 2.9%; 0.2 ≤ **Mo** ≤ 0.9%; **V** ≤ 0.4%.

Source: KUNITAKE, T. & KATOU, T. **Effect of Various Alloying Elements on Si-Cr-Mo-V Steel**. Tetsu-to-Hagané, 50:4, 1964, 666-668.

. Lee

$$A_{cm} = 224.4 + 992.4\,C - 465.1\,C^2 + 46.7\,Cr + 19.0\,C\,Cr - 6.1\,Cr^2 + 7.6\,Mn + 10.0\,Mo - 6.8\,Cr\,Mo - 6.9\,Ni + 3.7\,C\,Ni - 2.7\,Cr\,Ni + 0.8\,Ni^2 + 16.7\,Si$$

Notation:

A_{cm}: Upper Equilibrium Temperature Between Ferrite and Cementite [°C]
Alloy Content: [weight %]

Observations:

- Equation valid within the following alloy range: 0.2% ≤ **C** ≤ 0.7%; **Mn** ≤ 1.5%; **Si** ≤ 0.3%; **Ni** ≤ 2.8%; **Cr** ≤ 1.5%; **Mo** ≤ 0.6%.
- Regression coefficient r^2 = 0.998837; precision interval: ± 3°C.

Source: LEE, S.J. & LEE, Y.K. *Thermodynamic Formula for the A_{cm} Temperature of Low Alloy Steels*. **ISIJ International**, 47:5, May 2007, 769-774.

. Park

$$Ac_3 = 955 - 350\ C - 25\ Mn + 51\ Si + 106\ Nb + 100\ Ti + 68\ Al - 11\ Cr - 33\ Ni - 16\ Cu + 67\ Mo$$

Notation:

Ac_3: Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
Alloy Content: [weight %]

Observations:

- Formula specifically developed for TRIP steels.

Source: PARK, S.H. et alii. *Development of Ductile Ultra-High Strength Hot Rolled Steels*. **Posco Technical Report**, 1996, 50-128.

. Roberts

$$Ae_3 = 910 - 25\ Mn - 11\ Cr - 20\ Cu + 60\ Si + 700\ P - 250\ Al - F_n$$

Notation:

Ae_3: Upper Equilibrium Temperature Between Ferrite and Austenite [°C]
Alloy Content: [weight %]
F_n: value defined according to the table below:

C	F_n
0.05	24
0.10	48
0.15	64
0.20	80
0.25	93
0.30	106
0.35	117
0.40	128

Source: ROBERTS, W.L.: **Flat Processing of Steel**; Marcel Dekker Inc., New York, 1988.

. Trzaska (I)

$$Ac_1 = 739 - 22.8\,C - 6.8\,Mn + 18.2\,Si + 11.7\,Cr - 15\,Ni - 6.4\,Mo - 5\,V - 28\,Cu$$

$$Ac_3 = 937.3 - 224.5\sqrt{C} - 17\,Mn + 34\,Si - 14\,Ni + 21.6\,Mo + 41.8\,V - 20\,Cu$$

Notation:

Ac_1: Lower Temperature of the Ferrite-Austenite Field During Heating [°C]
Ac_3: Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
Alloy Content: [weight %]

Source: TRZASKA, J. et alii. *Modelling of CCT Diagrams for Engineering and Constructional Steels*. **Journal of Materials Processing Technology**, 192-193, 2007, 504-510.

. Trzaska (II)

$$Ac_1 = 742 - 29\,C - 14\,Mn + 13\,Si + 16\,Cr - 17\,Ni - 16\,Mo + 45\,V + 36\,Cu$$

$$Ac_3 = 925 - 219\,\sqrt{C} - 7\,Mn + 39\,Si - 16\,Ni + 13\,Mo + 97\,V$$

Notation:

Ac_3: Lower Temperature of the Ferrite-Austenite Field During Heating [°C]
Ac_3: Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
Alloy Content: [weight %]

Observations:

- Equation valid within the following alloy range: 0.06% ≤ C ≤ 0.68%; 0.13 ≤ Mn ≤ 2.04%; 0.12 ≤ Si ≤ 1.75%; Ni ≤ 3.85%; Cr ≤ 2.30%; Mo ≤ 1.05%; V ≤ 0.38%; Cu ≤ 0.38.
- Additional validity limitations: Mn + Cr ≤ 3.6; Mn + Cr + Ni ≤ 5.6; Cr + Ni ≤ 5.3; Mn + Ni ≤ 4.5
- Ac_1 statistical parameters: regression coefficient: $r^2 = 0.61$; standard error: ± 15.55°C.
- Ac_3 statistical parameters: regression coefficient: $r^2 = 0.75$; standard error: ± 17.80°C.

Source: TRZASKA, J. *Calculation of Critical Temperatures by Empirical Formulae*. **Archives of Metallurgy and Materials**, 61:2B, 2016, 981-986.

- **Austenite Grain Size After Heating**

. Lee

$$d = 76671 \exp\left(- \frac{89098 + 3581\,C + 1211\,Ni + 1443\,Cr + 4031\,Mo}{R\,T} \right) t^{0.211}$$

Notation:

d: Austenite Grain Size [μm]
R: Universal Gas Constant, 8.314 J/mol.K
T: Austenitizing Temperature [K]
t: Austenitizing Time [s]

Observations:

- Equation valid under the following alloy range: 0.15% ≤ C ≤ 0.41%; 0.73% ≤ Mn ≤ 0.85; 0.20% ≤ Si ≤ 0.25%; Ni ≤ 1.80%; Cr ≤ 1.45%; Mo ≤ 0.45.

Source: LEE, S.J. et alii.: *Prediction of Austenite Grain Growth During Austenitization of Low Alloy Steels.* **Materials and Design**, 29, 2008, 1840-1844.

- **Austenite Hardenability: Critical Diameter**
-

. Dearden & O'Neill

$$D_i = 6 \times \exp\left[7.1 \times \left(C + \frac{Mn}{5.87} + \frac{Mo}{3.13} + \frac{Cr}{6.28} + \frac{Si}{18} + \frac{Ni}{15}\right)\right]$$

Notation:

D_i: Critical Diameter [mm]
Alloy Content: [weight %]

Source: DEARDEN, J & O'NEIL, H.: *A Guide to the Selection and Welding of Low Alloy Structural Steels*. **Transactions of the Institute of Welding**, 3, Oct. 1940, 203-214.

- **Austenite Hardenability: Martensite Formation Critical Cooling Rate**

. Gang

$$V_{c100} = e^{5.3 - 2.53\,C - 0.16\,Si - 0.82\,Mn - 0.95\,Cr - 1.87\,Mo - 160\,B}$$

Notation:

V_{c100}: Critical Cooling Rate to Obtain 100% Martensite Structure [°C/s]
Alloy Content: [weight %]

Source: GANG, L. et alii. *High Strength Steel with Yield Strength of 800 MPa and Production Method Therefor*. **International Patent WO2016095720A1**, 2016, 19 p.

. Ueno

For B Steels (free B ≥ 5 ppm):

$$V_{c90} = 10^{2.94 - 0.75\,\beta}$$

$$\beta = 2.7\,C + 0.4\,Si + Mn + 0.45\,Ni + 0.8\,Cr + 2\,Mo$$

For B-free Steels:

$$V_{c90} = 10^{3.69 - 0.75\,\beta'}$$

$$\beta' = 2.7\,C + 0.4\,Si + Mn + 0.45\,Ni + 0.8\,Cr + Mo$$

Notation:

V_{c90}: Critical Cooling Rate to Obtain 90% Martensite Structure [°C/s]
Alloy Content: [weight %]

Source: UENO, M. & ITOH, K.: *New Empirical Formula for Estimation of Hardenability from Chemical Compositions*. **Tetsu-to-Hagané**, 74:6, 1988, 1073-1080.

- **Austenite No-Recrystallization Temperature**

. Bai 1996

For interpass times less than or equal 12.5 s:

$$T_{nr} = \left[88.1\log([Nb] + 0.31[Ti] + 0.15[Al]) + 1156\right]\varepsilon^{-0.12}\,\dot{\varepsilon}^{\,-0.01}\,t^{-0.1}$$

For interpass times greater 12.5 s and less than or equal 30.0 s:

$$T_{nr} = \left[63.5\log([Nb][C]) + 885\right]\varepsilon^{-0.12}\,\dot{\varepsilon}^{\,-0.01}\,t^{0.04}$$

For interpass times less than 10 s:

$$\lambda = 3.52\log(\varepsilon) + 5.3 - (3.10\log(\varepsilon) + 4.95)\frac{T}{T_{nr}}$$

$$RLT = T_{nr}\,\frac{3.52\log(\varepsilon) + 5.30}{3.10\log(\varepsilon) + 4.95}$$

$$RST = T_{nr}\,\frac{3.52\log(\varepsilon) + 4.70}{3.10\log(\varepsilon) + 4.95}$$

For interpass times equal or greater than 10 s and equal or less than 30 s:

$$\lambda = 3.52\log(\varepsilon) + 6.4 - (3.10\log(\varepsilon) + 5.93)\frac{T}{T_{nr}}$$

$$RLT = T_{nr}\,\frac{3.52\log(\varepsilon)+6.40}{3.10\log(\varepsilon)+5.93}$$

$$RST = T_{nr}\,\frac{3.52\log(\varepsilon)+5.80}{3.10\log(\varepsilon)+5.93}$$

Notation:

T_{nr}: No-Recrystallization Temperature [°C].
Alloy Content: [weight %]
ε: Pass True Strain.
$\dot{\varepsilon}$: Pass Strain Rate [s^{-1}].
t: Interpass Time [s].
λ: Residual Strain Ratio Between Consecutive Passes.
T: Pass Temperature [°C].

Observations:

- ε must be higher than 0.15 for calculation of RLT.

Source: BAI, D. et alii.: *Static Recrystallization of Nb and Nb-B Steel Under Continuous Cooling Conditions.* **Transactions of the ISIJ**, 36:8, August 1996, 1084-1093.

. Bai 2011

$$RLT = 174\log\left[Nb\left(C+\frac{12}{14}N_{eff}\right)\right]+1444$$

$$RST = RLT - 75$$

$$N_{eff} = N - \frac{14}{48} Ti$$

Notation:

RLT: Recrystallization Limit Temperature [°C]: full austenite recrystallization between deformation steps is no longer possible below this temperature.

RST: Recrystallization Stop Temperature [°C]: austenite recrystallization stops completely below this temperature.

Alloy Content: [weight %]

Observations:

- $N_{eff} \geq 0$.

Source: BAI, D. et alii.: *Development of Discrete X80 Line Pipe Plate at SSAB Americas.* In: **International Symposium on the Recent Developments in Plate Steels**. Proceedings. Association for Iron and Steel Technology, Winter Park, 2011, p. 13-22.

. Boratto

$$T_{nr} = 887 + 464\,C + (6445\,Nb - 644\sqrt{Nb}) + (732\,V - 230\sqrt{V}) + 890\,Ti + 363\,Al - 357\,Si$$

Notation:

T_{nr}: Temperature of No-Recrystallization [°C]. Maximum temperature at which austenite recrystallizes completetly between two deformation passes.

Alloy Content: [weight %]

Observations:

- Equation valid under the following alloy range: 0.04 ≤ C ≤ 0.17%; 0.41 ≤ Mn ≤ 1.90%; 0.15 ≤ Si ≤ 0.50%; 0.002 ≤ Al ≤ 0.650%; Nb ≤ 0.060%; V ≤ 0.120%; Ti ≤ 0.110%; Cr ≤ 0.67%; Ni ≤ 0.45%.

Source: BORATTO, F. et alii.: *Effect of Chemical Composition on Critical Temperatures of Microalloyed Steels.* In: **International Conference on Physical Metallurgy of Thermomechanical Processing of Steels and Other Metals - THERMEC '88**. Proceedings. Iron and Steel Institute of Japan, Tokyo, 1988, p. 383-390.

. Fletcher

$$T_{nr} = 849 - 349\ C + 676\ \sqrt{Nb} + 337\ V$$

Observations:

- $r^2 = 0.72$

Notation:

T_{nr}: Temperature of No-Recrystallization [°C]. Maximum temperature at which austenite recrystallizes completetly between two deformation passes.

Alloy Content: [weight %].

Source: FLETCHER, M.: *Meta-Analysis of Temperature of No-Recrystallization Measurements: Determining New Empirical Models Based on Composition and Strain*. In: **Austenite Processing Symposium**, Arcelor-Mittal Internal Symposium, 2008, 1-14.

. Fletcher-Bai

$$T_{nr} = 203 - 310\ C + 657\ \sqrt{Nb} - 149\ \sqrt{V} + 683\ e^{-0.36\ \varepsilon}$$

Notation:

T_{nr}: Temperature of No-Recrystallization [°C]. Maximum temperature at which austenite recrystallizes completetly between two deformation passes.

Alloy Content: [weight %].

ε: True Strain.

Source: FLETCHER, M.: *Meta-Analysis of Temperature of No-Recrystallization Measurements: Determining New Empirical Models Based on Composition and Strain*. In: **Austenite Processing Symposium**, Arcelor-Mittal Internal Symposium, 2008, 1-14.

. Militzer

$$T_{nr} = 893 + 910\ Nb$$

Notation:

T_{nr}: Temperature of No-Recrystallization [°C]. Maximum temperature at which austenite recrystallizes completetly between two deformation passes.

Alloy Content: [weight %]

Observations:

- Equation valid for 0.020% ≤ Nb ≤ 0.090%.

Source: MILITZER, M.: *Modelling of Microstructure Evolution and Properties of Low-Carbon Steels.* **Acta Metallurgica Sinica (English Letters)**, 13:2, April 2000, 574-580.

- **Austenite Solubility Products**

. General Formula

$$\log_{10} k_s = \log_{10} \frac{(a_M)^m\ (a_X)^n}{a_{M_mX_n}} = \log_{10} \frac{[M]^m\ [X]^n}{[MX]} = -\left(\frac{Q}{2.303\,RT}\right) + \frac{C}{2.303} = -\frac{A}{T} + B$$

Notation:

M_mX_n: Precipitate Considered for Calculation
a_i: Activity
M, X: Alloy Contents [weight %]
T: Temperature [K]
C: Constant
A, B: Constants of the Solubility Product, given in the table below:

Precipitate	A	B	Source
AlN	7060	1.55	Narita
	6770	1.03	Irvine
	7750	1.80	Ashby
BN	13970	5.24	Fountain
$Cr_{23}C_6$	7375	5.36	Ashby
Mo_2C	7375	5.00	Ashby
NbC	9290	4.37	Johansen
	7290	3.04	Meyer
	7900	3.42	Narita

	7510	2.96	Nordberg
	6700	2.06	Palmiere
NbCN	5860	1.54	Meyer
	6770	2.26	Ashby
$NbC_{0.87}$	7520	3.11	Ashby
	7700	3.18	Mori
NbN	8500	2.80	Narita
	10230	4.04	Smith
	10800	3.70	Ashby
TiC	7000	2.75	Irvine
TiN	15020	3.82	Narita
	8000	0.32	Ashby
VC	9500	6.72	Narita
V_4C_3	8000	5.36	Ashby
VN	8330	3.46	Irvine
	7070	2.27	Irvine
ZrC	8464	4.26	Narita
ZrN	16007	4.26	Narita

Observations:

- $\mathbf{a_{AmBn}}$ is equal to one if the precipitate is pure.
- $\mathbf{a_{AmBn} \leq 1}$ if there is co-precipitation with another element.
- The product $\mathbf{[M]^m[X]^n}$ (that is, $\mathbf{k_s}$) defines the graphical boundary of solubilization in a graph [M] x [X].

Sources:

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- SMITH, R.P. The Solubility of Niobium (Columbium) Nitride in Gamma Iron. **Transactions of the Metallurgical Society of AIME**, 224:2, 1962, 190-191.

- Values compiled by Rajindra Clement Ratnapuli and Fúlvio Siciliano from assorted references when not specified above.

. Dong

$$\log[Nb]\left[C+\frac{12}{14}N\right]=3.14+0.35[Si]-0.91[Mn]+\frac{1371[Mn]-923[Si]-8049}{T}$$

Notation:

T: Temperature [K]

Alloy Content: [weight %]

Source: DONG, J.X. et alii.: *Effect of Silicon on the Kinetics of Nb(C,N) Precipitation during the Hot Working of Nb-bearing Steels*. **ISIJ International**, 40:6, June 2000, 613-618.

. Irvine

$$\log[Nb]\left[C+\frac{12}{14}N\right]=2.26-\frac{6770}{T}$$

Source: IRVINE, K.J. et alii. *Grain-Refined C-Mn Steels*. **Journal of the Iron and Steel Institute**, 205:2, Feb. 1967, 161-182.

. Ashby

$$\log[V]\left[N\right]=3.46+0.12\,Mn-\frac{8330}{T}$$

Notation:

T: Temperature [K]

Alloy Content: [weight %]

Source: ASHBY, M.F. & EASTERLING, K.E. *A First Report on Diagrams for Grain Growth in Welds*. **Acta Metallurgica**, 30, 1982, 1969-1978.

. Koyama

$$\log [Nb]\,[C] = \frac{-7970}{T} + 3.31 + \left(\frac{1371}{T} - 0.900\right)[Mn] - \left(\frac{75}{T} - 0.0504\right)[Mn]^2$$

$$\log [Nb]\,[C] = \frac{-7970}{T} + 3.31 + \left(\frac{735}{T} - 0.348\right)[Si]$$

$$\log [Nb]\,[C] = \frac{-7970}{T} + 3.31 + \left(\frac{1113}{T} - 0.691\right)[Cr] - \left(\frac{38}{T} - 0.0228\right)[Cr]^2$$

$$\log [Nb]\,[C] = \frac{-7970}{T} + 3.31 + \left(\frac{148}{T} - 0.094\right)[Ni] - \left(\frac{8.5}{T} - 0.0068\right)[Ni]^2$$

Notation:

T: Temperature [K]
Alloy Content: [weight %]

Source: KOYAMA, S. et alii.: *Effects of Mn, Si, Cr and Ni on the Solution and Precipitation of Niobium Carbide in Iron Austenite*. **Journal of the Japanese Institute of Metals**, 35:11, November 1971, 1089-1094.

$$\log [Nb]\,[N] = \frac{-8500}{T} + 2.89 + \left(\frac{1085}{T} - 0.68\right)[Mn] - \left(\frac{48}{T} + 0.032\right)[Mn]^2$$

$$\log [Nb]\,[N] = \frac{-8500}{T} + 2.89 + \left(\frac{1900}{T} - 1.103\right)[Si]$$

$$\log [Nb]\,[N] = \frac{-8500}{T} + 2.89 + \left(\frac{1290}{T} - 0.77\right)[Cr] - \left(\frac{51}{T} + 0.034\right)[Cr]^2$$

$$\log [Nb]\,[N] = \frac{-8500}{T} + 2.89 + \left(\frac{694}{T} - 0.44\right)[Ni] - \left(\frac{29}{T} - 0.0178\right)[Ni]^2$$

Notation:

T: Temperature [K]
Alloy Content: [weight %]

Source: KOYAMA, S. et alii.: *The Effects of Mn, Si, Cr and Ni on the Reaction of Solution and Precipitation of Niobium Nitride in Iron Austenite*. **Journal of the Japanese Institute of Metals**, 35:7, July 1971, 698-705.

. Mori

$$\log [Nb]\,[N]^{0.65}\,[C]^{0.24} = 4.09 - \frac{10400}{T}$$

Notation:

T: Temperature [K]

Alloy Content: [weight %]

Source: MORI, T. et alii.: *Thermodynamic Behaviors of Niobium-Carbide-Nitride and Sulfide in Steel*. **Tetsu-to-Hagané**, 51:11, 1965, 2031-2011.

. Siciliano

$$\log[Nb]\left[C+\frac{12}{14}N\right]=2.26+\frac{838\,[Mn]^{0.246}-1730\,[Si]^{0.594}-6440}{T}$$

Notation:

T: Temperature [K]

Alloy Content: [weight %]

Source: SICILIANO JR., F..: *Mathematical Modeling of the Hot Strip Rolling of Nb Microalloyed Steels*. **Ph.D. Thesis**, McGill University, February 1999, 165 p.

- **Austenite Solubilization: Solid Solution**

. Uranga

Nb microalloyed steel

$$C_{sol} = -0.123 + 0.985\,C + 0.07804\,N - 0.07329\,Nb + 0.147\,e^{\left(\frac{-205.7}{T+271}\right)}$$

$$N_{sol} = -0.0054 + 0.00403\,C + 0.752\,N - 0.04245\,Nb + 0.0516\,e^{\left(\frac{-2610.8}{T+271}\right)}$$

$$Nb_{sol} = 0.0176 - 0.0116\,C - 1.172\,N_{eff} + 0.08878\,Nb + 60030\,e^{\left(\frac{-20587.3}{T+273}\right)}$$

NbTi microalloyed steel

$$C_{sol} = -0.348 + 0.884\,C + 0.374\,N - 0.0157\,Nb - 0.959\,Ti + 0.462\,e^{\left(\frac{-252}{T+273}\right)}$$

$$N_{sol} = -0.00168 + 0.00847\,C + 0.683\,N - 0.0185\,Nb - 0.197\,Ti + 0.0339\,e^{\left(\frac{-2846}{T+273}\right)}$$

$$Nb_{sol} = 0.0126 - 0.166\,C - 0.936\,N + 0.365\,Nb + 0.0241\,Ti + 38696\,e^{\left(\frac{-21250}{T+273}\right)}$$

$$Ti_{sol} = 0.001515 - 0.00337\,C - 0.304\,N - 0.00157\,Nb - 0.0914\,Ti + 943570\,e^{\left(\frac{-30377}{T+273}\right)}$$

Notation:

Alloy Content: [weight %]
T: Temperature [°C]

Observations:

- Equations fitted using Thermocalc predictions (TCFE6 Database).
- Useful temperature range: 900 – 1300°C.
- Useful composition range: 0.05% ≤ C ≤ 0.10%, 0.020% ≤ Nb ≤ 0.100%, 0.002% ≤ Ti ≤ 0.025% (except Nb microalloyed steels) and 0.0030 ≤ N ≤ 0.0120.

Source: URANGA, P. et alii. *Solubility Equations for Nb-Ti Microalloyed Steels*. **Technical Report**, CEIT, Donostia/San Sebastian, 2014, 4 p.

Ti Steel

$$Ti_{sol} = Ti - \frac{48}{14}\,N - \frac{48}{32}\,S$$

B Steel

$$B_{sol} = B - \frac{10.8}{14}\,N$$

- **Austenite Solubilization: Temperature**

. General Formula

$$T_d(^0C) = \frac{A}{B - \log_{10}(a_A)^m (a_B)^n} - 273$$

Notation:

A_mB_n: Precipitate considered for calculation
T_d: Solubilization temperature [°C]
a_x: Alloy content [weight %]
A, B: Constants of the solubility product, given in the table at the topic *Austenite Solubilization Products*.

. Chastukhin

$$T_d = \log_{10}([C]^{203.6}\,[Nb]^{231.6}) + 33.86\,[Si] - 1.48\,[Mn] - 2.73\,[Cr] + 17.1\left|\,3.61 - \frac{[Ti]}{[N]}\,\right| + 1657$$

Notation:

T_d: Dissolution temperature of NbCN [°C]
Alloy Content: [weight %]

Source: CHASTUKHIN, A.V. et alii. *Formation of Austenitic Structure During Heating of Slabs of Pipe Steels Microalloyed with Niobium*. **Metallurgist**, 59:7-8, November 2015, 581-589.

. Uranga - 1

$$T_d = \frac{-20587.3}{\ln\left(\frac{-0.0176 + 0.116\,[C] + 1.172\,N_{eff} + 0.9122\,Nb}{60030}\right)} - 273$$

$$N_{eff} = N - \frac{12}{48}Ti$$

Notation:

T_d: Dissolution temperature of NbCN [°C]
Alloy Content: [weight %]

Observations:

- $N_{eff} \geq 0$.
- Formula developed for NbTi microalloyed steels.

Source: URANGA, P. et alii. *Private Communication*. 2016.

. Uranga - 2

- $$\underline{T_d = \frac{-21250}{\ln((-0.0126 + 0.166 * C + 0.936 * N + 0.635 * Nb - 0.0241 * Ti) / 38696)} - 273}$$
-
-

Notation:

T_d: Dissolution temperature of NbCN [°C]
Alloy Content: [weight %]

Observations:

- Formula developed for NbTi microalloyed steels.

Source: URANGA, P. et alii. *Private Communication*. 2014.

- **Austenite Transformation Temperatures: Ferrite Start and Finish**

. Blás

$$Ar_3 = 903 - 328\,C - 102\,Mn + 116\,Nb - 0.909\,v$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Amount: [weight %]
v: Cooling Rate [°C/s]

Observations:

- This formula was determined using temperature data got from samples cooled directly from hot rolling experiments. Thus it includes the effects of hot forming over austenite decomposition.
- Useful range: 0.024-0.068% C, 0.27-0.39% Mn, 0.004-0.054% Al, 0.000-0.094% Nb, 0.0019-0.0072% N, 1.0-35°C/s
- r = 0.934; Root Mean Square Deviation = 5°C

Source: BLÁS, J.G. et alii: *Influência da Composição Química e da Velocidade de Resfriamento sobre o Ponto Ar_3 em Aços de Baixo C Microligados ao Nb.* In: **44° Congresso Anual da Associação Brasileira de Metais**, ABM, São Paulo, vol. 1, Outubro 1989, p. 11-29.

. Choo

$$Ar_3 = 925.95 - 494.74\,C - 64.8\,Mn - 10\,v^{0.5}$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Amount: [weight %]
v: Cooling Rate [°C/s]

Source: CHOO, S.D. & CHOO, W.Y.: *An Integrated Model for Microstructural Evolution and its Application to the Properties Prediction of Plate Steels.* In: **42nd Mechanical Working and Steel Processing**, Iron and Steel Society, Warrendale, 2000, p. 407-417.

. Choquet

$$Ar_3 = 902 - 527\,C - 62\,Mn + 60\,Si$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Amount: [weight %]

Observations:

- This formula was determined using data got from samples cooled directly from hot rolling experiments. Thus it includes the effects of hot forming over austenite decomposition.

Source: CHOQUET, P. et alii.: *Mathematical Model for Predictions of Austenite and Ferrite Microstructures in Hot Rolling Processes.* **IRSID Report**, St. Germain-en-Laye, 1985. 7 p.

. Gang

$$Ar_3 = 901 - 325\,C - 92\,Mn - 126\,Cr - 67\,Ni - 149\,Mo$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Content: [weight %]

Observations:

- The previous conditioning of the steel samples that supplied data for the deduction of this formula is unknown.

Source: GANG, L. et alii. *High Strength Steel with Yield Strength of 800 MPa and Production Method Therefor*. **International Patent WO2016095720A1**, 2016, 19 p.

. Higashi

$$Ar_3 = 901 - 325\,C - 92\,Mn + 33\,Si - 46\,(Ni + Cr + Cu + Mo)$$

Notation:
Ar_3: Ferrite Start Temperature [°C]
Alloy Content: [weight %]

Observations:
- The previous conditioning of the steel samples that supplied data for the deduction of this formula is unknown.

Source: HIGASHI, M. et alii. *Hot Stamping Molded Body Excellent in Strength and Hydrogen Embrittlement Resistance and Method for Producing Hot Stamping Molded Body*. **Japanese Patent JP5942841B2**, 2016, 7 p.

. Kariya

$$Ar_3 = 910 - 203\sqrt{C} - 30\,Mn + 44.7\,Si - 11\,Cr + 31.5\,Mo - 15.2\,Ni$$

Notation:
Ar_3: Ferrite Start Temperature [°C]
Alloy Amount: [weight %]

Source: KARIYA, N. **High Carbon Hot-Rolled Steel Sheet and Method for Production Thereof**. European Patent Application EP 2.103.697.A1, 23.09.2009, 15 p.

. Kotsar

$$Ar_3 = 879.2 - 94.24\,C - 21.13\,Si - 25.56\,Mn + 47.71\,Cr + 16.44\,Ni$$

$$Ar_1 = 729.2 - 9.24\,C + 12.13\,Si - 5.56\,Mn + 17.71\,Cr - 46.44\,Ni$$

Notation:
Ar_3: Ferrite Start Temperature [°C]
Ar_1: Ferrite Finish Temperature [°C]
Alloy Content: [weight %]

Observations:
- Ar_3 equation: Maximum Error = 85.5°C (11.9%)
- Ar_1 equation: Maximum Error = 35.7°C (4.7%)

Source: KOTSAR, S.L. et alii. *Tekhnologia Listoprocatnogo Proizvodstva (Technology of Sheet Rolling Production).* **Metallurgiya**, Moscow., 1997, 277 p.

. Leysen

$$Ar_3 = 901 - 830\,C - 61\,Mn + 71\,Si + 362\,P$$

Notation:
Ar_3: Ferrite Start Temperature [°C]
Alloy Content: [weight %]

Source: Leysen, F. et alii. *Development of Steels with Low Ar_3-Temperature for Austenitic Rolled Strip Applications.* **3rd International Conference on Recrystallization and Related Phenomena**, Monterey, 1996, 437-443.

. Lotter

$$Ar_3 = 834.8 - 251\,C - 103.2\,Mn + 60.5\,Si - 69.7\,Cr - 105.5\,Mo + 204.5\,Nb + 202\,Ti + 9.0\,B + 11.86\sinh^{-1}\left(\Delta t_{8/5}\right)$$

Observations:

- This formula was determined using data got from samples submitted to a normalizing rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; final cooling.
- r = 0.92; Root Mean Square Deviation = 21°C

$$Ar_3 = 884.2 - 331\,C - 98.7\,Mn + 65.2\,Si - 75.9\,Cr - 97.4\,Cu - 76.7\,Mo - 322.6\,Nb + 158\,Ti + 290\,V + 9.28\sinh^{-1}\left(\Delta t_{8/5}\right)$$

Observations:

- This formula was determined using data got from samples submitted to a thermomechanical rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; 35% @800°C; final cooling.
- r = 0.88; Root Mean Square Deviation = 23°C

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Amount: [weight %]
$\Delta t_{8/5}$: Time Between 800°C and 500°C [s]

Source: LOTTER, U. *Aufstellung von Regressionsgleichungen zur Beschreibung des Umwandlungsverhaltens beim thermomechanischen Walzen.* Technische Bericht, Thyssen Stahl, Duisburg, 1988, 136 p.

. Lutsenko

$$Ar_3 = 913.7 - 207.13\,C - 46.6\,Mn + 110.54\,Cr + 108.1\,N$$

$$Ar_1 = 741.7 - 7.13\,C - 14.09\,Mn + 16.26\,Si + 11.54\,Cr - 49.69\,Ni$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Ar_1: Ferrite Finish Temperature [°C]
Alloy Amount: [weight %]

Source: LUTSENKO, A. et alii. *The Definition and Use of Technological Reserves – An Effective Way to Improve the Production Technology of Rolled Metal*. Vestnik of Nosov Magnitogorsk State Technical University, 5, 2013, 70-4.

. Miettinen

Carbon Steels:

$$Ar_3 = 888.26 - 581.45\,C + 76.66\,Si - 55.10\,Mn + 553.59\,C\,C - 211.01\,C\,Si - 31.13\,C\,Mn + 4.04\,GS - 6.37\sqrt{CR} - 4.88\ln(CR)$$

Alloy Steels:

$$Ar_3 = 892.15 - 523.35\,C + 86.50\,Si - 65.88\,Mn - 46.85\,Cr + 21.82\,Mo - 45.17\,Ni + 405.05\,C\,C - 180.54\,C\,Si - 3.65\,C\,Mn + 92.20\,C\,Cr - 69.66\,C\,Mo + 37.50\,C\,Ni + 2.43\,GS - 5.18\sqrt{CR} - 6.78\ln(CR)$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Amount: [weight %]

GS = Austenite Grain Size [ASTM units]
CR = Cooling Rate [°C/s]

Observations:
- Carbon Steels: r = 0.97, average error = 14°C. Useful range: 0.05 ≤ C ≤ 0.60%, 0.30 ≤ Mn ≤ 2.00, 0.15 ≤ Si ≤ 0.70%, Cr < 0.5%, 0.2% Mo, 0.5% Ni, 1 ≤ ASTM GS ≤ 10 and CR ≥ 0.05°C/s.
- Alloy Steels: r = 0.95, average error = 16°C. Useful range: 0.05 ≤ C ≤ 0.60%, 0.30 ≤ Mn ≤ 2.00, 0.15 ≤ Si ≤ 0.70%, Cr ≤ 2.0%, Mo ≤ 0.5%, Ni ≤ 3.5%, 1 ≤ ASTM GS ≤ 10 and CR ≥ 0.05°C/s.

Source: MIETTINEN, J. *Simple Semiempirical Model for Prediction of Austenite Decomposition and Related Heat Release During Cooling of Low Alloyed Steels*. **Ironmaking and Steelmaking**, 23:4, 1996, 346-356.

. Mintz

First Proposal:

$$Ar_3 = 833.6 - 190.6\,C - 67.4\,Mn + 1522\,S - 2296\,N_{ti} - 1532\,Nb + 7.91\,d^{-1/2} - 0.117\,CR$$

$$N_{ti} = N_t - \frac{Ti}{3.5}$$

Observations:
- Useful range: 0.04-0.75% C, 0.30-1.60% Mn, 0.02-0.49% Si, 0.014-0.085% Al, 0.00-0.31% Nb, 0.004-0.008% N, 0.003-0.032% S, d: 0.070-0.950 mm, CR: 25-200°C/min
- r = 0.949; Root Mean Square Deviation = 15.9°C. The coefficients for N_{ti} (82.2%), d (87.4%) and CR (92.8%) were not significant for a 95% minimum confidence level.
- The unexpected positive effect of S can be due to enhanced nucleation of ferrite at sulphides.

Second Proposal:

$$Ar_3 = 868 - 181\,C - 75.8\,Mn + 1086\,S - 3799\,N_{ti} - 1767\,Nb - 0.0933\,CR$$

$$Ar_3 = 862 - 182\,C - 76.1\,Mn + 1121\,S - 1804\,Nb + 1168\,Ti - 2852\,N_t - 0.0084\,CR$$

$$N_{ti} = N_t - \frac{Ti}{3.5}$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Amount: [weight %]
N_t: Total Nitrogen Content [weight %]
d = Austenite Grain Diameter [mm]
N_t: Total Nitrogen Content [weight %]
CR = Cooling Rate [°C/min]

Observations:

- This formula was determined using temperature data got from non-hot deformed samples and includes TRIP steels.
- Useful range: 0.04-0.75% C, 0.31-2.52% Mn, 0.01-1.22% Si, 0.00-1.55% Al, 0.000-0.042% Nb, 0.0012-0.014% N, 0.002-0.110% P, 0.001-0.032% S, d: 0.1-1.0 mm, CR: 10-200°C/min
- r = 0.939; Root Mean Square Deviation = 18.1°C.

Third Proposal: Specific for TRIP Steels

$$Ar_3 = 870 - 586\,C - 630\,P + 18.1\,Al - 8151\,Nb$$

Observations:

- Useful range: 0.12-0.22% C, 1.02-2.52% Mn, 0.006-1.05% Si, 0.022-1.55% Al, 0.000-0.026% Nb, 0.0012-0.014% N, 0.010-0.110% P, 0.001-0.0056% S, d: 0.1-1.0 mm, CR: 30 and (mostly) 60°C/min

- r = 0.957; Root Mean Square Deviation = 16.6°C.

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Amount: [weight %]
N_t: Total Nitrogen Content [weight %]
d = Austenite Grain Diameter [mm]
CR = Cooling Rate [°C/min]

Source: MINTZ, B. et alii. *Regression Equation for Ar_3 Temperature for Coarse Grained as Cast Steels*. **Ironmaking and Steelmaking**, 38:3, March 2011, 197-203.

. Okamoto

$$Ar_3 = 901 - 325\,C - 92\,Mn + 33\,Si + 287\,P + 40\,Al - 20\,Cr$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Amount: [weight %]

Source: NAKAMURA, T. & ESAKA, K. *Development of Hot Rolled Steel Sheet with High r Value*. **International Conference on Physical Metallurgy of Thermomechanical Processing of Steels and Other Metals - THERMEC '88**, ISIJ, Tokyo, 1988, 644-651.

. Ouchi

$$Ar_3 = 910 - 310\,C - 80\,Mn - 20\,Cu - 15\,Cr - 55\,Ni - 80\,Mo + 0{,}35\,(h - 8)$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Content: [weight %]
h: Plate Thickness [mm]

Observations:
- This formula was determined using temperature data got from samples of Nb microalloyed steels cooled directly from hot rolling experiments. Thus it includes the effects of hot forming over austenite decomposition.

Source: OUCHI, C. et alii. *The Effect of Hot Rolling Condition and Chemical Composition on the Onset Temperature of Gamma-Alpha Transformation After Hot Rolling*. **Transactions of the ISIJ**, 22:3, March 1982, 214-222.

. Pickering

$$Ar_3 = 910 - 230\,C - 21\,Mn - 15\,Ni + 32\,Mo + 45\,Si + 13\,W + 104\,V$$

Notation:
Ar_3: Ferrite Start Temperature [°C]
Alloy Content: [weight %]

Observations:
- Applicable to Plain C Steels.

Source: PICKERING, F.B.: *Steels: Metallurgical Principles*. In: **Encyclopedia of Materials Science and Engineering**, vol. 6, The MIT Press, Cambridge, 1986.

. Proprietary #1

$$Ar_3 = 879.4 - 516.1\,C - 65.7\,Mn + 38.0\,Si + 274.7\,P$$

Notation:
Ar_3: Ferrite Start Temperature [°C]
Alloy Content: [weight %]

Source: Unknown.

. Proprietary #2

$$Ar_1 = 706.4 - 350.4\,C - 118.2\,Mn$$

Notation:
Ar_1: Ferrite Finish Temperature [°C]
Alloy Content: [weight %]

Observations:
- The previous conditioning of the steel samples that supplied data for the deduction of this formula is unknown.
- Samples cooled at 20°C/s.

Source: Unknown.

. Salganik

$$Ar_3 = 735.6 + 180.1\,(C + Cr) + 1206.9\,(S + P) - 10.9\,(Si + Mn + Ni + Cu + Mo) + 755.3\,(Al + N) - 328.8\,(V + Nb + Ti)$$

$$Ar_1 = 576.8 + 195.7\,(C + Cr) + 3022.6\,(S + P) - 17.5\,(Si + Mn + Ni + Cu + Mo) + 1040\,(Al + N) - 440.6\,(V + Nb + Ti)$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Ar_1: Ferrite Finish Temperature [°C]
Alloy Content: [weight %]

Observations:
- Equations fitted with data from 10 steels
- Formulas valid within the following range: 0.060% ≤ C ≤ 0.130%, 0.59% ≤ Mn ≤ 1.65%, 0.24% ≤ Si ≤ 0.46%, 0.007% ≤ P ≤ 0.012%, 0.001% ≤ S ≤ 0.025%, 0.020% ≤ Cr ≤ 0.070%, 0.02% ≤ Ni ≤ 0.23%, 0.03% ≤ Cu ≤ 0.22%, 0.008% ≤ Al ≤ 0.059%, Nb ≤ 0.048%, Ti ≤ 0.023%, V ≤ 0.048%, Mo ≤ 0.19%, 0.0020% ≤ N ≤ 0.007%
- Ar_3 equation: $r^2 = 0.955$, Standard Error of Estimate = 4.64%
- Ar_1 equation: $r^2 = 0.976$, Standard Error of Estimate = 4.01%
- Applicable to low C microalloyed steels for large diameter pipes.

Source: SALGANIK, V.M. et alii: *Analysis of Structural and Phase Transformations in Low-Alloy Steels Based on Dilatometric Studies*. **Metallurgist**, 59:9-10, January 2016, 766-773.

. Santos

$$Ar_3 = 874.44 - 512.0465\ C - 40.915\ Mn + 23.075\ Si + 567.126\ C^2 - 199.551\ C\ Mn + 265.797\ C\ Si + 4.148\ A - 1.03\ \sqrt{CR} - 11.334\ \ln(CR)$$

$$A = -2 \ln\left(\frac{0.002\ d_\gamma}{\ln(2)}\right)$$

Notation:
Ar_3: Ferrite Start Temperature [°C]
Alloy Content: [weight %]
A: Austenite Grain Size [ASTM units]
CR: Continuous Cooling Rate [°C/s]
d_γ: Austenite Grain Size [µm]

Observations:
- Equation fitted with data from 94 points; $r^2 = 0.9888$
- Applicable to plain C Steels.

Source: SANTOS, A.A.: *Previsão das Temperaturas Críticas de Decomposição da Austenita em Ferrita e Perlita Durante Resfriamento Contínuo*. In: **41° Seminário de Laminação – Processos e Produtos Laminados e Revestidos**, Associação Brasileira de Metalurgia e Materiais, Joinville, 2004, 10 p.

. Schacht

$$Ar_3' = 811 - 255\ C - 7\ Mn + 19\ Si$$

$$Ar_3 = Ar_3' - 19\ v_r^{0.481} - 0.5\ e^{\left(\frac{0.042\ d_\gamma + 7.8}{2.7402}\right)}$$

$$Ar_1 = 739 - 22\ C - 7\ Mn + 2\ Si$$

Notation:
Ar_3': Ferrite Start Temperature without Undercooling [°C]
Ar_3: Ferrite Start Temperature with Undercooling [°C]
Alloy Content: [weight %]
v_r: Continuous Cooling Rate [°C/s]
d_γ: Austenite Grain Size [µm]

Observations:
- Equation fitted with data from 94 points; $r^2 = 0.9888$
- Applicable to plain C Steels.

Source: SCHACHT, K. et alii.: *Material Models and their Capability for Process and Materials Properties Design in Different Forming Processes*. **Materials Science Forum**, 854, 2016, 174-182.

. Sekine

$$Ar_3 = 868 - 396\,C - 68.1\,Mn + 24.6\,Si - 36.1\,Ni - 24.8\,Cr - 20.7\,Cu$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Content: [weight %]

Observations:

- This formula was determined using temperature data got from samples cooled directly from hot rolling experiments. Thus it includes the effects of hot forming over austenite decomposition.
- Precision: ±13°C.

Source: TAMURA, I. et alii.: **Thermomechanical Processing of High-Strength Low-Alloy Steels**. Butterworths, London, 1988, 248 p.

. Shiga

$$Ar_3 = 910 - 273\,C - 74\,Mn - 56\,Ni - 16\,Cr - 9\,Mo - 5\,Cu$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Content: [weight %]

Observations:

- This formula was determined using temperature data got from samples cooled directly from hot rolling experiments. Thus it includes the effects of hot forming over austenite decomposition.

Source: SHIGA, C. et alii.: *Development of Large Diameter High Strength Line Pipes for Low Temperature Use*. **Kawasaki Steel Technical Report**, December 1981, 97-109.

. Trzaska (I)

$$Ar_3 = 857 - 257\ C - 69\ Mn + 23\ Si - 38\ Ni - 20\ Cr - 20\ Mo + 34\ V + 26\ Cu + 0.07\ T_A - 17\ v_R^{0.25}$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Content: [weight %]
T_A: Austenitizing Temperature [°C]
v_R: Cooling Rate [°C/min]

Observations:

- Formula valid within the following range: 0.21% ≤ C ≤ 0.68%, 0.28% ≤ Mn ≤ 2.00%, 0.13% ≤ Si ≤ 1.90%, Cr ≤ 2.5%, Ni ≤ 3.85%, Mo ≤ 1.05%, V ≤ 0.38% and Cu ≤ 0.38%.
- Error = 19.5°C, r = 0.86.

Source: TRZASKA, J.: *Empirical Formulae for the Calculation of Austenite Supercooled Transformation Temperatures*. **Archives of Metallurgy and Materials**, 60:1, 2015, 181-185.

. Trzaska (II)

$$Ar_3 = 1.375\ Ac_3 - 2.3\ CR - 339$$

$$Ac_3 = 937.3 - 224.5\ \sqrt{C} - 17\ Mn + 34\ Si - 14\ Ni + 21.6\ Mo + 41.8\ V - 20\ Cu$$

Observations:

- Standard Error = 11°C, r^2 = 0.990.

$$Ar_1 = 202.75\,CE^{-0.25} - 2.3\,CR + 402$$

$$CE = C + \frac{Mn}{6} + \frac{Cr + Mo + V}{5} + \frac{Cu + Ni}{15}$$

Observations:

- Standard Error = 11°C, r^2 = 0.986.

Notation:

Ac_3: Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
Ar_3: Ferrite Start Temperature [°C]
Ar_1: Ferrite Finish Temperature [°C]
Alloy Content: [weight %]
CE: Equivalent Carbon [weight %]
CR: Cooling Rate [°C/s]

Observations:

- Formulas valid within the following range: 0.029% ≤ C ≤ 0.73%, 0.27% ≤ Mn ≤ 0.66%, 0.03% ≤ Si ≤ 0.22%, 2°C/s ≤ CR ≤ 8°C/s

Source: TRZASKA, J.: *The Combined Effect of Chemical Composition and Cooling Rate on Transformation Temperatures of Hypoeutectoid Steels*. **Kovove Materiali**, 56, 2018, 163-170.

. Vanderschueren

$$Ar_3 = 865 - 280\ C - 60\ Mn + 60\ Si + 600\ P$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Content: [weight %]

Source: VANDERSCHUEREN, D. et alii.: *Influence of Transformation Induced Recrystallisation on Hot Rolling Textures of Low Carbon Steel Sheet*. **Materials Science and Technology**, 6:12, Dec. 1990, 1247-1250.

. Yuan

Non-Deformed Austenite:

$$Ar_3 = 370 \exp\left(-\frac{\sqrt{D_\gamma}}{6.7}\right) - 325\ CR^{0.1} - 5649\ Nb + 78194\ Nb^2 + 1019$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
CR: Continuous Cooling Rate [°C/s]
Nb: Niobium content [weight %]

Observations:

- This formula was determined using temperature data got from non-hot deformed samples.
- Base steel: 0.11% C, 1.20% Mn, 0.20% Si, 0.005% N. Useful range: 0.000-0.038% Nb, CR: 0.5-30°C/s.

Deformed Austenite:

$$Ar_3 = 370 \exp\left(-\frac{\sqrt{D_\gamma}}{6.7}\right) - 198\, CR^{0.1} - 6646\, Nb - 2327\, Nb^2 + 66\left(\frac{1}{t_{0.05}} + \Delta\varepsilon\right) + 830$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
CR: Continuous Cooling Rate [°C/s]
Nb: Niobium content [weight %]
$t_{0.05}$: Nb(CN) Precipitation Start Time [s]
$\Delta\varepsilon$: Residual Strain in Austenite

Observations:

- This formula was determined using temperature data got from samples cooled directly from hot rolling experiments. Thus it includes the effects of hot forming over austenite decomposition.
- Base steel: 0.11% C, 1.20% Mn, 0.20% Si, 0.005% N. Useful range: 0.000-0.038% Nb, CR: 0.5-30°C/s. See reference for details about the calculation of $t_{0.05}$ and $\Delta\varepsilon$, which requires external models.

Source: YUAN, X.Q. et alii.: *The Onset Temperatures of γ to α-Phase Transformation in Hot Deformed and Non-Deformed Nb Micro-Alloyed Steels*. **ISIJ International**, 46:4, Apr. 2006, 579-585.

. Záhumenský

$$Ar_3 = 914 - 6.85\, CR - 650\, C - 134\, Mn + 179\, Si$$

$$Ar_1 = 814 - 9.08\, CR - 532\, C - 121\, Mn + 165\, Si$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Ar_1: Ferrite Finish Temperature [°C]
CR: Continuous Cooling Rate [°C/s]

Alloy Content: [weight %]

Observations:

- This formula was determined using data got from CMn, HSLA, IF, DP and TRIP Steels.
- Maximum value of cooling rate: 15°C/s.
- Nb, Ti and Al contents were excluded from the equation due to statistical reasons.
- Ar_3: $r^2 = 0.91$
- Ar_1: $r^2 = 0.90$

Source: ZÁHUMENSKY, P. et alii: *Austenite-Ferrite Transformation Temperature Regression Equations for Low Carbon Steels with Cooling Rate Account*. **IOP Conference Series: Materials Science and Engineering**, 283:012024, 2017, 9 p.

. Zhao

$$M_a = 820 - 603.76\,C + 247.13\,C^2 - 66.24\,Mn - 55.72\,Ni + 3.97\,Ni^2 - 0.151\,Ni^3 - 31.10\,Cr + 2.348\,Cr^2 - 24.29\,Mo - 31.88\,Cu - 0.196\,Co + \\ + 0.165\,Co^2 - 0.00255\,Co^3 - 28.01\,Ru$$

Notation:

M_a: Massive Ferrite Start Temperature [°C]
Alloy Content: [weight %]

Source: ZHAO, J.: *Continuous Cooling Transformations in Steels*. **Materials Science and Technology**, 8:11, November 1992, 997-1002.

- **Austenite Transformation Temperatures: Pearlite Start and Finish**

. Miettinen

Carbon Steels:

$$P_s = 776.89 - 399.13\ C - 53.65\ Si - 49.80\ Mn + 501.46\ C\ C + 39.63\ C\ Si - 3.09\ C\ Mn + 3.35\ GS - 7.05\sqrt{CR} - 3.22 \ln(CR)$$

Alloy Steels:

$$P_s = 770.75 - 352.88\ C - 18.19\ Si - 72.46\ Mn - 6.46\ Cr - 16.03\ Mo - 40.53\ Ni + 391.71\ C\ C - 9.37\ C\ Si + 40.02\ C\ Mn + 23.42\ C\ Cr - 90.83\ C\ Mo + 21.84\ C\ Ni + 3.49\ GS - 3.63\sqrt{CR} - 7.71 \ln(CR)$$

Observations:

- Carbon Steels: r = 0.93, average error = 15°C.
- Alloy Steels: r = 0.89, average error = 18°C.

Carbon Steels:

$$P_f = 738.88 - 375.73\ C + 59.04\ Si - 85.65\ Mn + 468.34\ C\ C - 185.53\ C\ S + 68.02\ C\ Mn + 3.35\ GS - 12.41\sqrt{CR} - 5.28 \ln(CR)$$

Alloy Steels:

$$P_s = 729.77 - 353.70\ C + 97.38\ Si - 104.28\ Mn - 11.54\ Cr - 39.85\ Mo - 44.17\ Ni + 405.21\ C\ C - 237.62\ C\ Si + 102.20\ C\ Mn + 32.89\ C\ Cr - 96.79\ C\ Mo + 27.23\ C\ Ni + 3.49\ GS - 7.52\sqrt{CR} - 10.61 \ln(CR)$$

Observations:
- Carbon Steels: r = 0.85, average error = 10°C.
- Alloy Steels: r = 0.78, average error = 10°C.

Notation:
Ar_3: Ferrite Start Temperature [°C]
Alloy Amount: [weight %]
GS = Austenite Grain Size [ASTM units]
CR = Cooling Rate [°C/s]

Observations:
- Carbon Steels. Useful range: 0.05 ≤ C ≤ 0.60%, 0.30 ≤ Mn ≤ 2.00, 0.15 ≤ Si ≤ 0.70%, Cr < 0.5%, 0.2% Mo, 0.5% Ni, 1 ≤ ASTM GS ≤ 10 and CR ≥ 0.05°C/s.
- Alloy Steels. Useful range: 0.05 ≤ C ≤ 0.60%, 0.30 ≤ Mn ≤ 2.00, 0.15 ≤ Si ≤ 0.70%, Cr ≤ 2.0%, Mo ≤ 0.5%, Ni ≤ 3.5%, 1 ≤ ASTM GS ≤ 10 and CR ≥ 0.05°C/s.

Source: MIETTINEN, J. *Simple Semiempirical Model for Prediction of Austenite Decomposition and Related Heat Release During Cooling of Low Alloyed Steels*. **Ironmaking and Steelmaking**, 23:4, 1996, 346-356.

. Trzaska

$$P_s = 780 - 30\,C - 65\,Mn + 24\,Si - 29\,Ni - 26\,Mo - 21\,Cu - 17\,v_R^{0.25}$$

Notation:
Ar_3: Ferrite Start Temperature [°C]
Alloy Content: [weight %]
v_R: Cooling Rate [°C/min]

Observations:

- Formula valid within the following range: 0.21% ≤ C ≤ 0.68%, 0.28% ≤ Mn ≤ 2.00%, 0.13% ≤ Si ≤ 1.90%, Cr ≤ 2.5%, Ni ≤ 3.85%, Mo ≤ 1.05%, V ≤ 0.38% and Cu ≤ 0.38%.
- Error = 19.4°C, r = 0.80.

Source: TRZASKA, J.: *Empirical Formulae for the Calculation of Austenite Supercooled Transformation Temperatures*. **Archives of Metallurgy and Materials**, 60:1, 2015, 181-185.

- **Austenite Transformation Temperatures: Bainite Start and Finish**

. Bodnar #1

$$B_s = 844 - 597\ C - 63\ Mn - 16\ Ni - 78\ Cr$$

Notation:
B_s: Bainite Start Temperature [°C]
Alloy Amount: [weight %]

Source: BODNAR, R.L. et alii. *Effects of Mn, Si and Purity on the Design of 3.5NiCrMoV, 1CrMoV and 2.25Cr-1Mo Bainitic Alloy Steel*. **Metallurgical Transactions A**, 20A, 1989, 1445-1460.

. Bodnar #2

$$B_s = 719 - 127\ C - 50\ Mn - 31\ Ni - 27\ Cr - 61\ Mo$$

Notation:
B_s: Bainite Start Temperature [°C]
Alloy Amount: [weight %]

Observations:
- Equation developed using data got from continuous cooled samples.

Source: BODNAR, R.L. et alii. *The Design of an Improved High-Temperature 1%Cr-Mo-V Rotor Steel*. In: **Superclean Rotor Steels**, Pergamon Press, 1991, 331-401.

. Kang

$$B_s = 634.8 - 193.1\,C + 102.4\,C^2 - 31.2\,Mn - 4.6\,Si - 18.6\,Ni - 32.4\,Cr - 15.6\,Mo + 10.36\ln\left(d_\gamma\right)$$

Notation:

B_s: Bainite Start Temperature [°C]
d_γ: Austenite Grain Size [microns]
Alloy Amount: [weight %]

Observations:

- Reliable chemical composition range: 0.10-1.00% C, 0.17-1.91% Mn, 0.40% Si max, 2.10% Ni max, 2.16% Cr max, 1.96% Mo max.
- Reliable prior austenite grain size range: 1.62-6.70 microns

Source: KANG, S. et alii. *Prediction of Bainite Start Temperature in Alloy Steels with Different Grain Sizes*. **ISIJ International**, 54:4, April 2014, p. 997-999.

. Kirkaldy

$$B_s = 656 - 57.7\,C - 35\,Mn - 75\,Si - 15.3\,Ni - 34\,Cr - 41.2\,Mo$$

Notation:

B_s: Bainite Start Temperature [°C]
Alloy Amount: [weight %]

Observations:

- This is a modification of Steven & Haynes' formula using isothermal transformation diagrams determined for low and high alloy steels produced by U.S. Steel.

Source: KIRKALDY, J.S. et alii. *Prediction of Microstructure and Hardenability in Low Alloy Steels*. In: **Phase Transformations in Ferrous Alloys**, AIME, Philadelphia, 1983, 125-148.

. Kunitake

$$B_s = 732 - 202\,C - 85\,Mn + 216\,Si - 37\,Ni - 47\,Cr - 39\,Mo$$

Notation:

B_s: Bainite Start Temperature [°C]
Alloy Amount: [weight %]

Observations:

- This author concluded that the measured B_s temperature for steels with a greater Ni or Cr content is much higher than that predicted by Steven & Haynes.
- Reliable chemical composition range: 0.11~0.56% C, 0.34~1.49% Mn, 0.14~0.40% Si, 0.07~1.99% Mo, 0.14~4.80% Cr, 0.23~4.33% Ni.
- Error = 10.5°C; correlation coefficient $r^2 = 0.97$.

Source: KUNITAKE, T. & OKADA, Y. *The Estimation of Bainite Transformation Temperatures in Steels by Empirical Formulas*. **Tetsu-to-Hagané**, 84:2, February 1998, 137-141.

. Leach

$$WB_s = 850 - 206\,C - 78\,Mn - 33\,Ni - 70\,Cr - 75\,Mo - 61\,Si$$

Notation:

WB_s: Widmanstätten Ferrite/Bainite Start Temperature [°C]
Alloy Amount: [weight %]

Observations:

- RMS Error = 70-76°C.

Source: LEACH, L. et alii. *Use of Fe-C Information as Reference for Alloying Effects on B_s*. **Metallurgical and Materials Transactions A**, 50A:10, October 2019, 4531-4540.

. Lee #1

$$B_s = 984.4 - 361.9\ C + 261.9\ C^2 - 28.3\ Mn + 43.7\ Si$$

Notation:

B_s: Bainite Start Temperature [K]
Alloy Amount: [weight %]

Observations:

- Formula specifically developed for TRIP steels.

Source: LEE, J.K. et alii. *Prediction of Tensile Deformation Behaviour of Formable Hot Rolled Steels*. **Posco Technical Research Laboratories Report**, Pohang, 1999.

. Lee #2

$$B_s = 745 - 110\ C - 59\ Mn - 39\ Ni - 68\ Cr - 106\ Mo + 17\ Mn\ Ni + 6\ Cr^2 + 29\ Mo^2$$

Notation:

B_s: Bainite Start Temperature [°C]
Alloy Amount: [weight %]

Observations:

- This formula is based in the equations of Steven & Haynes, Kirkaldy and Kunitake & Okada, as well data from many time-temperature diagrams for several steels, including low alloy steels and steels with Ni and Cr

contents up to 4.5%, which were published in the **Atlas of Time-Temperature Diagrams for Iron and Steels** by G.F. Vander Voort through ASM International, Metals Park, in 1991.
- Reliable chemical composition range: 0.10-0.80% C, 0.26-1.63% Mn, 0.13-0.67% Si, 0.00-1.96% Mo, 0.00-4.48% Cr, 0.00-4.34% Ni.

Source: LEE, Y.K. et alii. *Empirical Formula of Isothermal Bainite Start Temperature of Steels*. **Journal of Materials Science Letters**, 21:16, 2002, 1253-122.

. Li

$$B_s = 637 - 58\,C - 35\,Mn - 15\,Ni - 34\,Cr - 41\,Mo$$

Notation:

B_s: Bainite Start Temperature [°C]
Alloy Amount: [weight %]

Observations:
- This formula is a modification of Kirkaldy's B_s equation.
- It assumes that Si amount is constant and equal to 0.25%, as most low alloy steels exhibit a content of this alloy element in this order of magnitude.
- Reliable chemical composition range: 0.20-0.41% C, 0.31-1.01% Mn, 0.10-0.28% Si, 0.00-0.44% Mo, 0.02-0.98% Cr, 0.02-3.04% Ni, 0.05-0.11% Cu.

Source: LI, M. et alii. *A Computational Model for the Prediction of Steel Hardenability*. **Metallurgical and Materials Transactions B**, 29:6, June 1998, 661-672.

. Lotter

$$B_S = 809.8 - 558\,C - 96.4\,Mn + 38.7\,Si - 111.1\,Cr - 76.3\,Mo - 723.2\,Nb - 530\,Ti + 6.12\,\sinh^{-1}\left(\Delta t_{8/5}\right)$$

Observations:

- This formula was determined using data got from samples submitted to a normalizing rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; final cooling.
- r = 0.72; Root Mean Square Deviation = 29°C

$$B_s = 705.2 - 180\,C - 53.8\,Mn + 26.9\,Si - 92.9\,Cr - 72.0\,Mo - 248\,V + 3.96\,\sinh^{-1}\left(\Delta t_{8/5}\right)$$

Observations:

- This formula was determined using data got from samples submitted to a thermomechanical rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; 35% @800°C; final cooling.
- r = 0.64; Root Mean Square Deviation = 27°C

Notation:

B_s: Bainite Start Temperature [°C]
Alloy Amount: [weight %]
$\Delta t_{8/5}$: Time Between 800°C and 500°C [s]

Source: LOTTER, U. *Aufstellung von Regressionsgleichungen zur Beschreibung des Umwandlungsverhaltens beim thermomechanischen Walzen.* Technische Bericht, Thyssen Stahl, Duisburg, 1988, 136 p.

. Miettinen

Carbon Steels:

$$B_s = 537.86 - 132.17\,C + 138.04\,Si + 34.65\,Mn + 251.53\,C\,C - 831.99\,C\,Si - 7.46\,C\,Mn$$

Alloy Steels:

$$B_s = 578.19 - 100.75\,C + 81.32\,Si + 7.12\,Mn + 11.85\,Cr + 97.71\,Mo - 14.44\,Ni + 228.79\,C\,C - 449.69\,C\,Si - 57.11\,C\,Mn - 57.17\,C\,Cr - 166.88\,C\,Mo - 18.75\,C\,Ni$$

Notation:

B_s: Bainite Start Temperature [°C]
Alloy Amount: [weight %]

Observations:

- Carbon Steels: r = 0.88, average error = 18°C. Useful range: 0.05 ≤ C ≤ 0.60%, 0.30 ≤ Mn ≤ 2.00, 0.15 ≤ Si ≤ 0.70%, Cr < 0.5%, 0.2% Mo, 0.5% Ni, 1 ≤ ASTM GS ≤ 10 and CR ≥ 0.05°C/s.
- Alloy Steels: r = 0.82, average error = 23°C. Useful range: 0.05 ≤ C ≤ 0.60%, 0.30 ≤ Mn ≤ 2.00, 0.15 ≤ Si ≤ 0.70%, Cr ≤ 2.0%, Mo ≤ 0.5%, Ni ≤ 3.5%, 1 ≤ ASTM GS ≤ 10 and CR ≥ 0.05°C/s.

Source: MIETTINEN, J. *Simple Semiempirical Model for Prediction of Austenite Decomposition and Related Heat Release During Cooling of Low Alloyed Steels*. **Ironmaking and Steelmaking**, 23:4, 1996, 346-356.

. Steven & Haynes

$$B_s = 830 - 270\,C - 90\,Mn - 37\,Ni - 70\,Cr - 83\,Mo$$

$$B_{50} = B_s - 50$$

$$B_{100} = B_s - 120$$

Notation:

B_s: Bainite Start Temperature [°C]
Alloy Amount: [weight %]

B_x: Temperature Required for the Formation of *x%* of Bainite [°C]

Observations:
- Equation determined using data from isothermal transformation diagrams.
- Reliable chemical composition range: 0.10-0.55% C, 0.2-1.7% Mn, 0.0-1.0% Mo, 0.0-3.5% Cr, 0.0-5.0% Ni.

Source: STEVEN, W. & HAYNES, A.G. *The Temperature of Formation of Martensite and Bainite in Low Alloy Steels*. **Journal of the Iron and Steel Institute**, 183, 1956, 349-359.

. Suehiro

$$B_s = 718 - 425\,C - 42.5\,Mn$$

Notation:
B_s: Bainite Start Temperature [°C]
Alloy Amount: [weight %]

Source: SUEHIRO, M. et alii. *A Kinetic Model for Phase Transformation of Low C Steels during Continuous Cooling*. **Tetsu-to-Hagané**, 73:8, June 1987, 1026-1033.

. Takada

$$B_s = 1336 - 1446\,C - 62.3\,Mn - 36.5\,Si - 47.8\,Cr - 160\,V - 77.5\,Mo$$

Notation:
B_s: Bainite Start Temperature [K]
Alloy Amount: [weight %]

Observations:

- Formula developed specifically for forging steels.
- Reliable chemical composition range: 0.11-0.40% C, 0.50-2.52% Mn, 0.31-1.26% Si, 0.20-1.96% Cr.

Source: TAKADA, H. *Alloy Designing of High Strength Bainite Steels for Hot Forging*. **Tetsu-to-Hagané**, 88:9, September 2002, 534-538.

. Trzaska #1

$$B_S = 675 - 212\ C - 57\ Mn - 17\ Si - 29\ Ni - 49\ Cr - 60\ Mo - 94\ V + 0.056\ T_A - 1.6\ v_R^{0.25}$$

Notation:

Ar_3: Ferrite Start Temperature [°C]
Alloy Content: [weight %]
T_A: Austenitizing Temperature [°C]
v_R: Cooling Rate [°C/min]

Observations:

- Formula valid within the following range: 0.21% ≤ C ≤ 0.68%, 0.28% ≤ Mn ≤ 2.00%, 0.13% ≤ Si ≤ 1.90%, Cr ≤ 2.5%, Ni ≤ 3.85%, Mo ≤ 1.05%, V ≤ 0.38% and Cu ≤ 0.38%.
- Error = 30.6°C, r = 0.72.

Source: TRZASKA, J.: *Empirical Formulae for the Calculation of Austenite Supercooled Transformation Temperatures*. **Archives of Metallurgy and Materials**, 60:1, 2015, 181-185.

. Trzaska #2

$$B_s = 771 - 231.5\ C - 69\ Mn - 23\ Si - 58.5\ Cr - 31\ Ni - 55\ Mo - 41\ V$$

Notation:

B_s: Bainite Start Temperature [°C]
Alloy Content: [weight %]

Observations:
- Equation valid within the following alloy range: 0.06% ≤ C ≤ 0.68%; 0.13 ≤ Mn ≤ 2.04%; 0.12 ≤ Si ≤ 1.75%; Ni ≤ 3.85%; Cr ≤ 2.30%; Mo ≤ 1.05%; V ≤ 0.38%; Cu ≤ 0.38.
- Additional validity limitations: Mn + Cr ≤ 3.6; Mn + Cr + Ni ≤ 5.6; Cr + Ni ≤ 5.3; Mn + Ni ≤ 4.5
- Regression coefficient r^2 = 0.75; standard error: ± 27.53°C.

Source: TRZASKA, J. *Calculation of Critical Temperatures by Empirical Formulae*. **Archives of Metallurgy and Materials**, 61:2B, 2016, 981-986.

. van Bohemen

$$B_s = 839 - 86\,Mn - 23\,Si - 67\,Cr - 33\,Ni - 75\,Mo - 270\left[1 - \exp(-1.33\,C)\right]$$

Notation:
B_s: Bainite Start Temperature [°C]
Alloy Amount: [weight %]

Observation:
- Factors multiplying substitutional elements are less than 10% different from the factors found by Steven and Haynes.
- Correlation Coefficient **R^2** = 0.97, Standard Error of Estimate **σ** = 13°C.

Source: van Bohemen, S.M.C. *Bainite and Martensite Start Temperature Calculated with Exponential Carbon Dependence*. **Materials Science and Technology**, 28:4, April 2012, 487-495.

. Wang & Cao

$$B_s = -36.6\ Mn_{eq}$$

$$Mn_{eq} = Mn + 3.43\ Mo + 0.56\ Ni$$

Notation:

B_s: Bainite Start Temperature [°C]
Alloy Amount: [weight %]

Source: WANG, S. & KAO, P. *The Effect of Alloying Elements on the Structure and Mechanical Properties of ULCB Steels*. **J. of Materials Science**, 28, 1993, 5169-75.

. Zhao #1

$$B_s = 720 - 585.63\ C + 126.60\ C^2 - 91.68\ Mn + 7.82\ Mn^2 - 0.3378\ Mn^3 - 66.34\ Ni + 6.06\ Ni^2 - 0.232\ Ni^3 - 31.66\ Cr + 2.17\ Cr^2 - \\ - 42.37\ Mo + 9.16\ Co - 0.1255\ Co^2 + 0.000284\ Co^3 - 36.02\ Cu - 46.15\ Ru$$

Source: ZHAO, J.: *Continuous Cooling Transformations in Steels*. **Materials Science and Technology**, 8:11, Nov. 1992, 997-1002.

. Zhao #2

$$B_s = 630 - 45\ Mn - 40\ V - 35\ Si - 30\ Cr - 25\ Mo - 20\ Ni - 15\ W$$

Source: ZHAO, Z. et alii. *A New Empirical Formula for the Bainite Upper Temperature Limit of Steel*. **Journal of Materials Science**, 36, 2001, 5045-5056.

Notation:

B_s: Bainite Start Temperature [°C]
Alloy Amount: [weight %]

- **Austenite Transformation Temperatures: Martensite Start and Finish**

. Andrews

Linear

$$M_s = 539 - 423\ C - 30.4\ Mn - 17.7\ Ni - 12.1\ Cr - 7.5\ Mo$$

Non-Linear

$$M_s = 512 - 453\ C - 16.9\ Ni + 15\ Cr - 9.5\ Mo + 217\ C^2 - 71.5\ C\ Mn - 67.6\ C\ Cr$$

Notation:
M_s: Martensite Start Temperature [°C]
Alloy Content: [weight %]

Source: ANDREWS, K.W. *Empirical Formulae for the Calculation of Some Transformation Temperatures.* **Journal of the Iron and Steel Institute**, 203, Part 7, July 1965, 721-727.

. Capdevila

$$M_s = 764.2 - 302.6\ C - 30.6\ Mn - 16.6\ Ni - 8.9\ Cr + 2.4\ Mo - 11.3\ Cu + 8.58\ Co + 7.4\ W - 14.5\ Si$$

Notation:
M_s: Martensite Start Temperature [K]
Alloy Content: [weight %]

Observations:

- Equation valid for steels with chemical composition between the following limits: 0.001 ≤ C ≤ 1.65, Mn ≤ 3.76, Si ≤ 3.40, Cr ≤ 17.9, Ni ≤ 27.2, Mo ≤ 5.10, V ≤ 4.55, Co ≤ 30.0, Al ≤ 1.10, W ≤ 12.9, Cu ≤ 0.98, Nb ≤ 0.23, B ≤ 0,0010, 0.0001 ≤ N ≤ 0.060.

Source: CAPDEVILA, C. et alii. *Determination of M_s Temperature in Steels: A Bayesian Neural Network Model.* **ISIJ International**, 42:8, August 2002, 894-902.

. Carapella

$$M_s = 496.1\,(1 - 0.344\,C)\,(1 - 0.051\,Mn)\,(1 - 0.018\,Si)\,(1 - 0.025\,Ni)\,(1 - 0.039\,Cr)\,(1 - 0.016\,Mo)\,(1 - 0.010\,W)\,(1 - 0.067\,Co)$$

Notation:

M_s: Start Temperature of the Martensitic Transformation [°C]
Alloy Amount: [weight %]

Source: CARAPELLA, L.A. *Computing A^{11} or Ms (Transformation Temperature on Quenching)*, **Metal Progress**, 46, 1944, 108.

. Eichelman & Hull

$$M_s = 41.7\,(14.6 - Cr) + 5.6\,(8.9 - Ni) + 33.3\,(1.33 - Mn) + 27.8\,(0.47 - Si) + 1666.7\,(0.068 - C - N) - 17.8$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Content: [weight %]

Observations:

- Equation valid for 18-8 stainless steels.

Source: EICHELMAN, G.H. & HULL, F.C. *The Effects of Composition on the Temperature of Spontaneous Transformation of Austenite to Martensite in 18-8 Stainless Steels.* **Transactions of the American Society for Metals**, 45, 1953, p. 77-104.

. Eldis

$$M_s = 531 - 391.2\,C - 43.3\,Mn - 21.8\,Ni - 16.2\,Cr$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Content: [weight %]

Observations:

- Equation valid for steels with chemical composition between the following limits: 0.10~0.80% C; 0.35~1.80% Mn; < 1.50% Si; < 0.90% Mo; < 1.50% Cr; < 4.50% Ni.

Source: BARRALIS, J. & MAEDER, G. *Métallurgie Tome I: Métallurgie Physique.* **Collection Scientifique ENSAM**, 1982, 270 p.

. Finkler & Schirra

$$M_s = 635 - 474\left\{C + 0.86\left[N - 0.15\,(Nb + Zr)\right] - 0.066\,(Ta + Hf)\right\} - \left(33\,Mn + 17\,Cr + 17\,Ni + 21\,Mo + 39\,V + 11\,W\right)$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Content: [weight %]

Observations:

- Equation valid for high temperature martensitic steels with 8,0 to 14% Cr.

Source: FINKLER, H. & SCHIRRA, M. *Transformation Behavior of High Temperature Martensitic Steels with 8 to 14% Chromium.* **Steel Research**, 67:8, August 1986, p. 328-336.

. Grange & Stewart

$$M_s = 537.8 - 361.1\,C - 38.9\,(Mn + Cr) - 19.4\,Ni - 27.8\,Mo$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Source: GRANGE, R.A. & STEWART, H.M. *The Temperature Range of Martensite Formation*. **Transactions of the AIME**, 167, 1946, 467-490.

. Hougardy

$$M_s = 0.495\,M_{sjh} + 0.00095\,M_{sjh}^2 + 40$$

Notation:

M_s: Corrected Martensite Start Temperature [°C]
M_{sjh}: Martensite Temperature Start According to Jaffe & Hollomon [°C]
Alloy Amount: [weight %]

Observation:

- Correction of Jaffe & Hollomon equation considering several other similar equations already published.

$$V_M = 1 - \exp\left[-k\,(M_s - T)^q\right]$$

$$k = 0.36 \times 10^{-3} + 0.10 \times 10^{-4}\ M_s - 0.34 \times 10^{-6}\ M_s^2 + 0.32 \times 10^{-8}\ M_s^3 - 0.52 \times 10^{-11}\ M_s^4$$

$$q = 2.08 - 0.76 \times 10^{-2}\ M_s + 0.16 \times 10^{-4}\ M_s^2 - 0.90 \times 10^{-8}\ M_s^3$$

Notation:

V_M: Volume Fraction of Martensite
M_s: Temperature at Which 1% Martensite Forms [°C]
T: Temperature [°C]

Source: HOUGARDY, H.P. *Description and Control of Transformations in Technical Applications*. **Steel: A Handbook for Materials Research and Engineering – Volume 1: Fundamentals**, Springer-Verlag, Berlin, 1992, 167-200.

. Imai

$$M_S = 539 - 423\ C - 30.4\ Mn - 7.5\ Si + 30\ Al$$

Notation:

M_s: Start Temperature of the Martensitic Transformation [°C]
Alloy Amount: [weight %]

Observation:

- Formula specific for CMnAl TRIP steels.

Source: IMAI, N. et alii. *Effect of Alloying Element and Microstructure on Mechanical Properties of Low Alloy TRIP Steels*. **CAMP-ISIJ**, 8, 1995, 572-575.

. Jaffe & Hollomon

$$M_S = 550 - 350\,C - 40\,Mn - 35\,V - 20\,Cr - 17\,Ni - 10\,Cu - 10\,Mo - 8\,W + 15\,Co + 30\,Al$$

Notation:

M_s: Start Temperature of the Martensitic Transformation [°C]
Alloy Amount: [weight %]

Observation:

- Hougardy proposed a correction to this formula.

Source: GRANGE, R.A. & STEWART, H.M. *Hardenability and Quench Cracking*. **Transactions of the AIME**, 167, 1946, 617-646.

. Kunitake

$$M_s = 560.5 - 407.3\,C - 37.8\,Mn - 14.8\,Cr - 19.5\,Ni - 4.5\,Mo - 7.3\,Si - 20.5\,Cu$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Source: KUNITAKE, T. **Prediction of Ac_1, Ac_3 and M_s Temperatures by Empirical Formulas**. Journal of the Japan Society for Heat Treatment, 41, 2001, 164-169.

. Lee & Park

$$M_s = 475.9 - 335.1\,C - 34.5\,Mn - 1.3\,Si - 15.5\,Ni - 13.1\,Cr - 10.7\,Mo - 9.6\,Cu + 11.67\,\ln(d_\gamma)$$

Notation:

M_s: Martensite Start Temperature [°C]

d_γ: Austenite Grain Size [microns]
Alloy Amount: [weight %]

Source: LEE, S.J. & PARK, K.S. **Prediction of Martensite Start Temperature in Alloy Steels with Different Grain Sizes**. Metallurgical and Materials Transactions A, 44A:8, August 2013, p. 3423-3427.

. Lee & Van Tyne

$$V_M = 1 - \exp\left[-K_{LV}\,(M_s - T)^{n_{LV}}\right]$$

$$K_{LV} = 0.0231 - 0.0105\,C - 0.0017\,Ni + 0.0074\,Cr - 0.0193\,Mo$$

$$n_{LV} = 1.4304 - 1.1836\,C + 0.7527\,C^2 + 0.0258\,Ni - 0.0739\,Cr + 0.3108\,Mo$$

Notation:
V_M: Volume Fraction of Martensite
M_s: Martensite Start Temperature [K]
T: Temperature [K]
Alloy Amount: [weight %]

Observation:
- Start Temperature of Martensitic Transformation Calculated According to Capdevila.

Source: LEE, S.J. & VAN TYNE, C.J. *A Kinetics Model for Martensite Transformation in Plain C and Low-Alloyed Steels*. **Metallurgical and Materials Transactions A**, 43A:12, February 2012, 423-427.

. Li

$$M_s = 540 - 420\,C - 35\,Mn - 12\,Cr - 20\,Ni - 21\,Mo - 10.5\,Si - 10.5\,W + 20\,Al + 140\,V$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Source: LI, C. et alii. **Computation of M_s Temperature in Carbon Equivalence Method**. Journal of Liaoning Technology University, 17, 1998, 293-298.

. Liu

$$M_s = 550 - 361\,C - 39\,Mn - 35\,V - 20\,Cr - 17\,Ni - 10\,Cu - 5\,Mo - 5\,W + 16\,Co + 30\,Al$$

$$M_s = 538 - 317\,C - 33\,Mn - 28\,Cr - 17\,Ni - 11\,Si - 11\,Mo - 11\,W$$

. C < 0.05%

$$M_s = 550 - 350\,C - 45\,Mn - 30\,Cr - 20\,Ni - 16\,Mo - 5\,Si - 8\,W + 6\,Co + 15\,Al - 35\,(V + Nb + Zr + Ti)$$

. C > 0.05%

$$M_s = 525 - 350\,(C - 0.05) - 45\,Mn - 30\,Cr - 20\,Ni - 16\,Mo - 5\,Si - 8\,W + 6\,Co + 15\,Al - 35\,(V + Nb + Zr + Ti)$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Source: LIU, C. et alii. **A New Empirical Formula for M_s Temperature in Pure Iron and Ultra Low Carbon Alloy Steels**. Journal of Materials Processing Technology, 113:1-3, 2001, 556-562.

. Lotter

$$M_S = 558.3 - 312\,C - 49.0\,Mn - 24.9\,Cr - 649\,Ti$$

Observations:

- This formula was determined using data got from samples submitted to a normalizing rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; final cooling.
- r = 0.94; Standard Error of Deviation = 9.1°C

$$M_s = 452.6 - 245\,C - 8.1\,Mn - 36.7\,Cr - 71.8\,Cu - 146.7\,Mo - 120\,V$$

Observations:

- This formula was determined using data got from samples submitted to a thermomechanical rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; 35% @800°C; final cooling.
- r = 1.00; Standard Error of Deviation = 1.5°C

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]
$\Delta t_{8/5}$: Time Between 800°C and 500°C [s]

Source: LOTTER, U. *Aufstellung von Regressionsgleichungen zur Beschreibung des Umwandlungsverhaltens beim thermomechanischen Walzen.* Technische Bericht, Thyssen Stahl, Duisburg, 1988, 136 p.

. Mahieu

$$M_s = 539 - 423\,C - 30.4\,Mn - 7.5\,Si + 30.0\,Al$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Observation:

- Equation valid for TRIP steels with 0.91 ≤ Al ≤ 1.73%. Apparently it is a development of the Andrews formula.

Source: MAHIEU, J. et alii. *Phase Transformation and Mechanical Properties of Si-free CMnAl Transformation-Induced Plasticity-Aided Steel*. **Metallurgical and Materials Transactions A**, 33A:8, August 2002, 2573-2580.

. Miettinen

Carbon Steels:

$$M_s = 563.94 - 486.66\,C + 56.48\,S - 70.82\,Mn + 21.77\,C\,C - 33.52\,C\,Si + 85.55\,C\,Mn$$

Alloy Steels:

$$M_s = 562.93 - 489.48\,C + 59.74\,Si - 72.37\,Mn + 15.53\,Cr - 26.96\,Mo - 13.13\,Ni + 25.90\,C\,C - 39.43\,C\,Si + 90.07\,C\,Mn - 8.86\,C\,Cr + 73.21\,C\,Mo - 15.59\,C\,Ni$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Observations:

- Carbon Steels: r = 0.99, average error = 10°C. Useful range: 0.05 ≤ C ≤ 0.60%, 0.30 ≤ Mn ≤ 2.00, 0.15 ≤ Si ≤ 0.70%, Cr < 0.5%, 0.2% Mo, 0.5% Ni, 1 ≤ ASTM GS ≤ 10 and CR ≥ 0.05°C/s.
- Alloy Steels: r = 0.98, average error = 13°C. Useful range: 0.05 ≤ C ≤ 0.60%, 0.30 ≤ Mn ≤ 2.00, 0.15 ≤ Si ≤ 0.70%, Cr ≤ 2.0%, Mo ≤ 0.5%, Ni ≤ 3.5%, 1 ≤ ASTM GS ≤ 10 and CR ≥ 0.05°C/s.

Source: MIETTINEN, J. *Simple Semiempirical Model for Prediction of Austenite Decomposition and Related Heat Release During Cooling of Low Alloyed Steels*. **Ironmaking and Steelmaking**, 23:4, 1996, 346-356.

. Mikula & Wojnar

$$M_s = 635.02 - 549.82\,C - 85.441\,Mn - 68.967\,Si - 18.07\,Cr - 30{,}965\,Ni - 69.301\,Mo - 6.603\,V + 420.26\,Nb + 553.8\,Ti - 1746.5\,B$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Source: LIS, A.K. & LIS, J. *High Strength Hot Rolled and Aged Microalloyed 5% Ni Steel*. **Journal of Achievements in Materials and Manufacturing Engineering**, 18:1-2, September-October 2006, 37-42.

. Nehrenberg

$$M_s = 498.9 - 300\,C - 33.3\,Mn - 22.2\,Cr - 16.7\,Ni - 11.1\,(Si + Mo)$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Source: NEHRENBERG, A.E. In: *Contribution to Discussion on Grange and Stewart*. **Transactions of the AIME**, 167, 1946, 494-498.

. Payson & Savage

$$M_s = 498.9 - 316.7\ C - 33.3\ Mn - 27.8\ Cr - 16.7\ Ni - 11.1\ (Si + Mo + W)$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Source: PAYSON, P. & SAVAGE, C.H. *Martensite Reactions in Alloy Steels*. **Transactions A.S.M.**, 33, 1944, 261-280.

. Rowland & Lyle

$$M_s[°C] = 498.9 - 333.3\ C - 33.3\ Mn - 27.8\ Cr - 16.7\ Ni - 11.1\ (Si + Mo + W)$$

$$M_s[°F] = 930 - 600\ C - 60\ Mn - 50\ Cr - 30\ Ni - 20\ (Si + Mo + W)$$

$$M_{10}\ [°F] = M_s - 18$$

$$M_{50}\ [°F] = M_s - 85$$

$$M_{90}\ [°F] = M_s - 185$$

$$M_{100}\ [°F] = M_s - 387$$

Notation:

M_s: Martensite Start Temperature
M_x: Temperature Required for the Formation of *x%* of Martensite
Alloy Amount: [weight %]

Source: ROWLAND, E.S. & LYLE, S.R. *The Application of M_s Points to Case Depth Measurement*. **Transactions A.S.M.**, 37, 1946, 27-47.

. Shimamura

$$M_s = 517 - 300\,C - 11\,Si - 33\,Mn - 22\,Cr - 17\,Ni - 11\,Mo$$

Notation:
M_s: Martensite Start Temperature
Alloy Amount: [weight %]

Source: SHIMAMURA, J. et alii. *A Steel Plate for Line Pipe Having High Deformation Performance and a Method for Producing the Same*. **Japanese Patent JP5055774B2**, 2013, 16 p.

. Steven & Haynes

$$M_s = 561.1 - 473.9\,C - 33\,Mn - 16.7\,(Cr + Ni) - 21.1\,Mo$$

Notation:
M_s: Martensite Start Temperature [°C]

Source: STEVEN, W. & HAYNES, A.G. *The Temperature of Formation of Martensite and Bainite in Low Alloy Steels*. **Journal of the Iron and Steel Institute**, 183, 1956, 349-359.

. Sverdlin-Ness

$$M_s = 520 - 320\,C - 50\,Mn - 30\,Cr - 20\,(Ni + Mo) - 5\,(Cu + Si)$$

Notation:
M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Source: SVERDLIN, A.V. & NESS, A.R. *The Effects of Alloying Elements on the Heat Treatment of Steel*. In: **Steel Heat Treatment Handbook**, Marcel Dekker, New York, 1997, p. 45-91.

. Tamura

$$M_s = 550 - 361\,C - 39\,Mn - 20\,Cr - 17\,V - 17\,Ni - 10\,Cu - 5\,(Mo + W) + 15\,Co + 30\,Al$$

Notation:
M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Source: TAMURA, I. *Steel Material Study on the Strength*. **Nikkan Kogyo Shinbun Ltd.**, Tokyo, 1970, 40.

. Trzaska (I)

$$M_s = 411 - 328\,C - 13\,Mn - 9\,Ni - 3\,Cr - 16\,Mo + 34\,Cu + 6.7\,v_R^{0.25}$$

Notation:
M_s: Martensite Start Temperature [°C]
Alloy Content: [weight %]
v_R: Cooling Rate [°C/min]

Observations:

- Formula valid within the following range: 0.21% ≤ C ≤ 0.68%, 0.28% ≤ Mn ≤ 2.00%, 0.13% ≤ Si ≤ 1.90%, Cr ≤ 2.5%, Ni ≤ 3.85%, Mo ≤ 1.05%, V ≤ 0.38% and Cu ≤ 0.38%.
- Error = 20.5°C, r = 0.88.

Source: TRZASKA, J.: *Empirical Formulae for the Calculation of Austenite Supercooled Transformation Temperatures*. **Archives of Metallurgy and Materials**, 60:1, 2015, 181-185.

. Trzaska (II)

$$M_s = 541 - 401\,C - 36\,Mn - 10.5\,Si - 14\,Cr - 18\,Ni - 17\,Mo$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Content: [weight %]

Observations:

- Equation valid within the following alloy range: 0.06% ≤ C ≤ 0.68%; 0.13 ≤ Mn ≤ 2.04%; 0.12 ≤ Si ≤ 1.75%; Ni ≤ 3.85%; Cr ≤ 2.30%; Mo ≤ 1.05%; V ≤ 0.38%; Cu ≤ 0.38.
- Additional validity limitations: Mn + Cr ≤ 3.6; Mn + Cr + Ni ≤ 5.6; Cr + Ni ≤ 5.3; Mn + Ni ≤ 4.5
- Regression coefficient r^2 = 0.87; standard error = ± 19.99°C.

Source: TRZASKA, J. *Calculation of Critical Temperatures by Empirical Formulae*. **Archives of Metallurgy and Materials**, 61:2B, 2016, 981-986.

. van Bohemen

$$f = 1 - \exp\left[-\alpha_m\,(T_{KM} - T)\right]$$

$$T_{KM} = 462 - 273\,C - 26\,Mn - 13\,Cr - 16\,Ni - 30\,Mo$$

$$\alpha_m = 0.0224 - 0.0107\,C - 0.0007Mn - 0.00012\,Cr - 0.00005\,Ni - 0.0001\,Mo$$

Notation:

f: Volume Fraction of Martensite as a Function of Undercooling Below T_{KM} Temperature ($T_{KM} - T$)
T: Temperature [°C]
T_{KM}: Theoretical Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Observation:

- Formula based from Koistinen and Marburger equation.
- **α_m**: Standard error of estimate **σ** = 0.0014 K^{-1}; correlation coefficient **R^2** = 0.79.

Source: VAN BOHEMEN, S.M.C and SIETSMA, J. *Effect of Composition on Kinetics of Athermal Martensite Formation in Plain Carbon Steels*. **Materials Science and Technology**, 25:8, August 2009, 1009-1012.

$$M_s = 565 - 31\,Mn - 13\,Si - 10\,Cr - 18\,Ni - 12\,Mo - 600\left[1 - \exp(-0.96\,C)\right]$$

Notation:

M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Observation:

- Standard error of estimate **σ** = 13°C; correlation coefficient **r^2** = 0.95.

$$f = 1 - \exp\left[-\alpha_m\,(M_s - T)\right]$$

$$\alpha_m = 27.2 - 0.14\,Mn - 0.21\,Si - 0.11\,Cr - 0.08\,Ni - 0.05\,Mo - 19.8\left[1 - \exp(-1.56\,C)\right]$$

Notation:

f: Volume Fraction of Martensite as a Function of Undercooling Below M_s Temperature ($M_s - T$)
T: Temperature [°C]
M_s: Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Observation:

- Formula based from Koistinen and Marburger equation.
- **α_m**: Standard error of estimate **σ** = 0.005 K^{-1}; correlation coefficient **r^2** = 0.97.

Source: VAN BOHEMEN, S.M.C. *Bainite and Martensite Start Temperature Calculated with Exponential Carbon Dependence*. **Materials Science and Technology**, 28:4, April 2012, 487-495.

. Zhao

$$M_s^{TM} = 420 - 208.33\,C - 33.428\,Mn + 1.296\,Mn^2 - 0.02167\,Mn^3 - 16.08\,Ni + 0.7817\,Ni^2 - 0.02464\,Ni^3 - 2.473\,Cr + 30.00\,Mo + 12.86\,Co - 0.2654\,Co^2 + 0.001547\,Co^3 - 7.18\,Cu - 72.65\,N - 43.36\,N^2 - 16.28\,Ru + 1.72\,Ru^2 - 0.08117\,Ru^3$$

$$M_s^{LM} = 540 - 356.25\,C - 47.59\,Mn + 2.25\,Mn^2 - 0.0415\,Mn^3 - 24.56\,Ni + 1.36\,Ni^2 - 0.0384\,Ni^3 - 17.82\,Cr + 1.42\,Cr^2 + 17.50\,Mo + 21.87\,Co - 0.468\,Co^2 + 0.00296\,Co^3 - 16.52\,Cu - 260.64\,N - 17.66\,Ru$$

Notation:

M_s^{TM}: Twinned Martensite Start Temperature [°C]
M_s^{LM}: Lath Martensite Start Temperature [°C]
Alloy Amount: [weight %]

Observation:

- If $\mathbf{M_s^{LM}}$ is higher than $\mathbf{M_s^{TM}}$, both lath martensite and twinned martensite can be present in steel.
- However, if $\mathbf{M_s^{LM}}$ is lower than $\mathbf{M_s^{TM}}$, only twinned martensite can exist. This condition is fulfilled for some steels above a critical composition, which can be determined setting $\mathbf{M_s^{LM}} = \mathbf{M_s^{TM}}$.

Source: ZHAO, J.: *Continuous Cooling Transformations in Steels*. **Materials Science and Technology**, 8:11, Nov. 1992, 997-1002.

- **Carbon Equivalent – H.A.Z. Hardenability**

. Dearden & O'Neill

$$C_{EQ_Dearden} = C + \frac{Mn}{6} + \frac{Mo}{4} + \frac{Cr + V}{5} + \frac{Cu}{13} + \frac{Ni}{15} + \frac{P}{2}$$

Notation:

$C_{EQ_Dearden}$: Equivalent Carbon (Dearden) [%]
Alloy Content: [weight %]

Source: DEARDEN, J & O'NEIL, H.: *A Guide to the Selection and Welding of Low Alloy Structural Steels*. **Transactions of the Institute of Welding**, 3, October 1940, 203-214.

. Bastien

$$C_{EQ_Bastien} = C + \frac{Mn}{4{,}4} + \frac{Mo}{7{,}7} + \frac{Cr}{15{,}4} + \frac{Ni}{10{,}3}$$

$$\ln(CR_m) = 13{,}9 - 10{,}6\, C_{EQ_Bastien}$$

Notation:

$C_{EQ_Bastien}$: Equivalent Carbon (Bastien) [%]
Alloy Content: [weight %]
CR_m: Critical Cooling Rate at 700°C [°C/s], that is, minimum cooling rate that produces a fully martensitic structure)

Source: BASTIEN, P.G. et al: *Prediction of the Structure and Mechanical Properties in the Heat Affected Zone of High Yield Weldable Steels*. **Metal Construction and British Welding Journal**, 49, 1970, 9-14.

. IIW - International Institute of Welding

$$C_{EQ_IIW} = C + \frac{Mn}{6} + \frac{Cr + Mo + V}{5} + \frac{Cu + Ni}{15}$$

Notation:

C_{EQ_IIW}: Equivalent Carbon (IIW) [%]
Alloy Content: [weight %]

Source: YURIOKA, N.: *Physical Metallurgy of Steel Weldability*. **ISIJ International**, 41:6, June 2001, 566-570.

. Kihara

$$C_{EQ_Kihara} = C + \frac{Mn}{6} + \frac{Mo}{4} + \frac{Cr}{5} + \frac{V}{14} + \frac{Ni}{40} + \frac{Si}{24}$$

Notation:

C_{EQ_Kihara}: Equivalent Carbon (Kihara) [%]
Alloy Content: [weight %]

Source: KIHARA, H. et alii. **Technical Report of JRIM**, 1, 1959, 93.

. Shinozaki

$$C_{EQ_FBW} = C + \frac{Mn}{5} + \frac{Si}{15} + \frac{Cr}{9} + 7\,Nb\,(1-10C) + \frac{V\,(50\,C-1)}{3} + 1.3\,Ti\,(1-5\,C) + \frac{Mo\,(1-6\,C)}{2} + 29\,B\,(11\,C-1)$$

Notation:

C_{EQ_FBW}: Equivalent Carbon Designed Specifically for Flash Butt Welding [%]
Alloy Content: [weight %]

Source: SHINOZAKI, M. et alii.: *Effects of Chemical Composition and Structure of Hot Rolled High Strength Steel Sheets on the Formability of Flash Butt Welded Joints*. **Kawasaki Steel Technical Report**, 6, Sept. 1982, 21-30.

. Stout

$$C_{EQ_Stout} = 1000\, C \left(\frac{Mn}{6} + \frac{Cr + Mo}{10} + \frac{Ni}{20} + \frac{Cu}{40} \right)$$

Notation:
C_{EQ_Stout}: Equivalent Carbon (Kihara) [%]
Alloy Content: [weight %]

Source: STOUT, R.D. et alii. **Welding Journal Research Supplement**, 55, 1976, 89s-94s.

. Yurioka

$$C_{EQ_Yurioka} = C + \frac{Mn}{6} + \frac{Mo}{4} + \frac{Cr}{8} + \frac{Ni}{12} + \frac{Si}{24} + \frac{Cu}{15}$$

$$\log(t_m) = 10{,}6\, C_{EQ_Yurioka} - 4{,}8$$

Notation:
$C_{EQ_Yurioka}$: Equivalent Carbon (Yurioka) [%]
Alloy Content: [weight %]
t_m: Critical Cooling Time from 800 to 500°C [s] (that is, maximum cooling time that produces a fully martensitic structure)

Source: YURIOKA, N. et alii.: **Metal Construction**, 19, 1987, 217R.

- **Carbon Equivalent – Hydrogen Assisted Cold Cracking**

. Bersch & Koch (Hoesch)

$$C_{EQ_Bersch} = C + \frac{Mn + Si + Cr + Mo + V + Cu + Ni}{20}$$

Notation:

C_{EQ_Bersch}: Equivalent Carbon for Pipeline Steels [%]
Alloy Content: [weight %]

Observations:

- Formula deduced for pipeline steels

Source:

- BERSCH, B. et alii. *Weldability of Pipe Steels for Low Operating Temperatures*. **3R International**, 1, 1977.

- PATCHETT, B.M. et alii.: **Casti Metals Blue Book: Welding Filler Metals**. Casti Publishing Corp., Edmonton, February 1993, 608 p. (CD Edition).

. DNV

$$C_{EQ_DNV} = C + \frac{Mn}{10} + \frac{Si}{24} + \frac{Ni + Cu}{40} + \frac{Cr}{5} + \frac{V}{10} + \frac{Mo}{4}$$

Notation:

C_{EQ_DNV}: Equivalent Carbon (DNV) [%]
Alloy Content: [weight %]

Source: HANNERZ, N.E.: *The Influence of Si on the Weldability of Mild and High Tensile Structural Steels*. **IIW Document IX-1169-80**, 1980.

. Graville

$$C_{EQ_HSLA} = C + \frac{Mn}{16} - \frac{Ni}{50} + \frac{Cr}{23} + \frac{Mo}{7} + \frac{Nb}{5} + \frac{V}{9}$$

Notation:

C_{EQ_HSLA}: Equivalent Carbon (Uwer & Graville) [%]
Alloy Content: [weight %]

Observations:

- Formula deduced for pipeline steels

Source: GRAVILLE, B.A.: In: **Proc. Conf. on Welding of HSLA Structural Steels**, ASM, Materials Park, 1976.

. Ito & Bessyo (I)

$$P_{cm} = C + \frac{Si}{30} + \frac{Mn + Cu + Cr}{20} + \frac{Ni}{60} + \frac{Mo}{15} + \frac{V}{10} + 5B$$

Notation:

P_{cm}: Cracking Parameter [%]
Alloy Content: [weight %]

Observations:

- Formula deduced for pipeline steels with C < 0.15%
- This is the most popular formula for this kind of material.

- Equation valid under the following conditions: 0.07% ≤ C ≤ 0.22%; 0.40% ≤ Mn ≤ 1.40%; Si ≤ 0.60%; V ≤ 0.12%; Cr ≤ 1.20%; Ni ≤ 1.20%; Cu ≤ 0.50%, Mo ≤ 0.7%, B ≤ 0,005%.

Sources:

- ITO, Y. & BESSYO, K. *Cracking Parameter of High Strength Steels Related to Heat Affected Zone Cracking.* **Journal of the Japan Welding Society.**, 37:9, September 1968, 983-991.

- ITO, Y. & BESSYO, K. *Weldability Formula of High Strength Steels Related to Heat-Affected-Zone Cracking.* **The Sumitomo Search**, 1, 1969, 59-70.

. Ito & Bessyo (II)

$$P_c = C + \frac{Si}{30} + \frac{Mn + Cu + Cr}{20} + \frac{Mo}{15} + \frac{V}{10} + \frac{d}{600} + \frac{H}{60}$$

Notation:

P_c: Cracking Parameter [%]
Alloy Content: [weight %], except
H: Hydrogen amount in the weld metal, [cm³/100 g]
d: Plate Thickness, [mm]

Source: ITO, Y. & BESSYO, K.: *Weldability Formula of High Strength Steels*. **I.I.W. Document IX-576-68**, 1968.

. Mannesmann

$$C_{EQ_PLS} = C + \frac{Si}{25} + \frac{Mn + Cu}{16} + \frac{Cr}{20} + \frac{Ni}{60} + \frac{Mo}{40} + \frac{V}{15}$$

Notation:

C_{EQ_PLS}: Equivalent Carbon for Pipeline Steels [%]
Alloy Content: [weight %]

Observations:

- Formula deduced for pipeline steels
- A version of this formula divides V by 10

Sources:

- DUREN, C. & NIEDEROFF, K.: In: **Proc. on Welding and Performance of Pipeline**, TWI, London, 1986.

- HEISTERKAMP, F. et alii.: *Metallurgical Concept And Full-Scale Testing of High Toughness, H_2S Resistant 0.03%C - 0.10%Nb Steel*. **C.B.M.M. Report**, São Paulo, February 1993.

. Uwer & Hohne

$$C_{EQ_Uwer} = C + \frac{Mn}{10} + \frac{Cu}{20} + \frac{Ni}{40} + \frac{Cr}{20} + \frac{Mo}{10}$$

Notation:

C_{EQ_Uwer}: Equivalent Carbon (Uwer & Hohne) [%]
Alloy Content: [weight %]

Source: UWER, D. & HOHNE, H.: *Determination of Suitable Minimum Preheating Temperature for the Cold-Crack-Free Welding of Steels*. **IIW Document IX-1631-91**, 1991.

. Yurioka

$$C_{EQ_Yurioka} = C + A(C)\left(\frac{Mn}{6} + \frac{Si}{24} + \frac{Cr + Mo + V}{5} + \frac{Cu}{15} + \frac{Ni}{20} + \frac{Nb}{5} + 5\,B\right)$$

$$A(C) = 0.75 + 0.25 \tanh\left[20\,(C - 0.12)\right]$$

Notation:

$C_{EQ_Yurioka}$: Equivalent Carbon for Pipeline Steels [%]
Alloy Content: [weight %]

Observations:

- Formula for C-Mn and microalloyed pipeline steels
- This formula combines Carbon Equivalent equations from IIW and P_{cm}

Sources:

- YURIOKA, N.: *Physical Metallurgy of Steel Weldability*. **ISIJ International**, 41:6, June 2001, 566-570.

- PATCHETT, B.M. et alii.: **Casti Metals Blue Book: Welding Filler Metals**. Casti Publishing Corp., Edmonton, February 1993, 608 p. (CD Edition).

- **Carbon Equivalent – Peritectic Point**

. Blazek

$$C_A = 0.0896 + 0.0458\,Al - 0.0205\,Mn - 0.0077\,Si - 0.0223\,Al^2 - 0.0239\,Ni + 0.0106\,Mo + 0.0134\,V - 0.0032\,Cr + 0.00059\,Cr^2 + 0.0197\,W$$

$$C_B = 0.1967 + 0.036\,Al - 0.0316\,Mn - 0.0103\,Si + 0.1411\,Al^2 + 0.05\,Al\,Si - 0.0401\,Ni + 0.03255\,Mo + 0.0603\,V + 0.0024\,Cr + 0.00142\,Cr^2 - 0.00059\,Cr\,Ni + 0.0266\,W$$

Notation:

C_A: Maximum Carbon Solubility of the δ-Ferrite Phase [%].
C_B: Peritectic composition of the Fe-C phase diagram [%].
Alloy Content: [weight %]

Observations:

- Formula valid within the following ranges: Al ≤ 2.0%, Cr ≤ 18.3%, Mn ≤ 2.1%, Mo ≤ 2.2%, Si ≤ 2.05%, P ≤ 0.1%, S ≤ 0.15%, Cu ≤ 1.35%, Ni ≤ 10.3%, V ≤ 1.08%, Ti ≤ 0.22%, Sn ≤ 0.03%, Nb ≤ 0.075% and W ≤ 0.5%.
- C_A: $r^2 = 0.99$, RMS Error = 0.0053%.
- C_B: $r^2 = 0.98$, RMS Error = 0.0126%.
- This model fitted data generated by the Thermocalc software.

Source: BLAZEK, K.E. et alii. *Calculation of the Peritectic Range for Steel Alloys*. **Iron and Steel Technology**, July 2008, 80-85.

. Mills

$$C_{P_Mills} = C + 0.02\,Mn + 0.04\,Ni - 0.1\,Si - 0.04\,Cr - 0.1\,Mo$$

Notation:

C_{P_Mills}: Equivalent Carbon for Peritectic Point [%]
Alloy Content: [weight %]

Source: XU, J. et alii. *Effect of Elements on Peritectic Reaction in Molten Steel Based on Thermodynamic Analysis*. **ISIJ International**, 52:12, October 2012, 1856-1861.

. Miyake

$$C_{P_Inf} = f_1 - 0.10$$

$$C_{P_Sup} = f_2 + 0.05$$

$$f_1 = 0.0828\,Si - 0.0195\,Mn + 0.07398\,Al - 0.04614\,Ni + 0.02447\,Cr + 0.01851\,Mo + 0.090$$

$$f_2 = 0.2187\,Si - 0.03291\,Mn + 0.2017\,Al - 0.06715\,Ni + 0.04776\,Cr + 0.04601\,Mo + 0.173$$

Notation:
C_{P_Inf}: Lower Bound of Carbon Peritectic Content Range [%]
C_{P_Sup}: Upper Bound of Carbon Peritectic Content Range [%]
Alloy Content: [weight %]

Observations:
- Alloy elements contents are assumed to be 4.0% or less, excluding 0%.

Source: MIYAKE, T. et alii. *Method of Continuous Casting of High-Aluminum Steel and Mold Powder*. **U.S. Patent n° US 8,146,649 B2**, April 3, 2012, 13 p.

. Shepherd

$$C'_{\delta} = 0.0927 - 0.0151\ Mn + 0.00776\ Si^2 + 0.0565\ Al + 0.0143\ Al^2 + 0.00338\ Al^3 - 0.0170\ Mn\ Si - 0.0148\ Mn\ Al - 0.0574\ Si\ Al - \\ - 0.00848\ Mn\ Si\ Al - 0.00900\ (Si\ Al)^2 - 0.0121\ (Si + Al) - 0.000775\ Si^4 + 0.00128\ (Mn\ Si)^3 + 0.00119\ (Mn + Si + Al)^3 + \\ + 0.000913\ (Mn\ Al)^4 - 0.00193\ (Mn\ Si\ Al)^4 - 0.000341\ (Mn + Si + Al)^4 - 0.0425\ P^2 + 0.0549\ P + 0.1369\ S - 0.0135\ Cu - \\ - 0.4694\ N + 0.0036\ Sn^2 - 0.014\ Sn - 0.0256\ Nb - 0.0357\ Ti + 0.0113\ V - 0.0009\ Mo^2 + 0.0062\ Mo - 0.0016\ Cr - \\ - 0.0195\ Ni$$

$$C'_{\gamma} = 0.249 + 0.0673\ Si^2 + 0.177\ Al^2 - 0.0232\ Mn\ Si - 0.0116\ Mn\ Al + 0.140\ Al^5 - 0.105\ Si\ Al + 0.0214\ Mn\ Si\ Al + \\ + 0.0104\ (Mn\ Si)^2 - 0.0429\ (Si\ Al)^2 - 0.195\ Al^4 + 0.0441\ Mn^4 - 0.0269\ Mn^5 - 0.0242\ e^{Mn} - 0.0437\ e^{Si} + 0.0233\ (Si\ Al)^4 + \\ + 0.0152\ (Mn\ Si\ Al)^4 - 0.000721\ (Mn + Si + Al)^4 + 0.2651\ P + 0.5573\ S - 0.0174\ Cu - 0.585\ N + 0.0094\ Sn^2 - 0.0211\ Sn - \\ - 0.027\ Nb + 0.0377\ Ti^2 - 0.0463\ Ti + 0.042\ V - 0.0015\ Mo^2 + 0.0238\ Mo + 0.0024\ Cr^2 - 0.002\ Cr - 0.0349\ Ni$$

$$C'_{liq} = 0.746 - 0.0469\ Mn + 0.0305\ Si - 0.0265\ Si^2 + 0.0236\ Si^3 + 1.37\ Al - 1.21\ Al^2 + 1.70\ Al^3 - 0.771\ \sqrt{Al} + \\ + 0.0745\ \log_{10}(Al) - 0.0351\ Mn\ Si - 0.0560\ Mn\ Al - 0.249\ Si\ Al + 0.00571\ Mn\ Si\ Al - 0.00973\ (Mn \cdot Al)^2 - 1.07\ Al^4 + \\ + 0.321\ Al^5 + 0.00544\ (Mn + Al)^2 - 0.0338\ (Si\ Al)^3 + 0.1065\ P + 1.239\ S - 0.0621\ Cu - 0.8642\ N + 0.001\ Sn^2 - \\ - 0.0191\ Sn + 0.0051\ Ti^2 + 0.0386\ Ti + 0.0043\ V^2 + 0.0896\ V + 0.046\ Mo + 0.0056\ Cr^2 - 0.0178\ Cr - \\ - 0.0047\ Ni^2 - 0.0763\ Ni$$

Notation:

C'δ: Maximum Carbon Solubility of the δ-Ferrite Phase [%]
C'δ: Peritectic Composition [%]
C'liq: Maximum Carbon Solubility of the Liquid Phase [%]
Alloy Content: [weight %]

Observations:

- Range of Validity: Mn ≤ 2.0%, P ≤ 0.192%, S ≤ 0.070%, Si ≤ 1.5%, Cu ≤ 1.0%, Al ≤ 2.0%, N ≤ 0.1%, Sn ≤ 1.0%, Nb ≤ 0.5%, Ti ≤ 0.44%, V ≤ 1.0%, Mo ≤ 1.0%, Cr ≤ 0.5% and Ni ≤ 0.5%.
- Standard Deviation: $C'_{\delta} = 0.0002$, $C'_{\delta} = 0.0011$, $C'_{liq} = 0.0023$.
- This model fitted data generated by the FactSage software.

Source: SHEPERD, R. et alii. *Improved Determination of the Effect of Alloying Elements on the Peritectic Range in Low-Alloyed Cast Steel*. **Iron and Steel Technology**, October 2012, 77-85.

. Wolf

$$C_{P_Wolf} = C + 0.04\,Mn + 0.1\,Ni + 0.7\,N - 0.14\,Si - 0.04\,Cr - 0.1\,Mo - 0.4\,Ti$$

Notation:

C_{P_Wolf}: Equivalent Carbon for Peritectic Point [%]
Alloy Content: [weight %]

Source: WOLF, M.M. *Estimation of Crack Susceptibility for New Steel Grades*. In: **1st European Conference on Continuous Casting**, Florence, 1991, 2489-2499.

. Xu

$$C_{P_Xu} = 0.1763 + 0.0616\,Al - 2.5275\,S + 0.2652\,P - 0.0023\,Si - 0.0344\,Mn + 1.5250\,S\,Mn - 0.0210\,Si\,Mn$$

Notation:

C_{P_Xu}: Equivalent Carbon for Peritectic Point [%]
Alloy Content: [weight %]

Observations:

- Formula valid within the following ranges: Si ≤ 0.5%, Mn ≤ 1.5%, Al ≤ 0.06%, P ≤ 0.05%, S ≤ 0.015%.

- $r^2 = 0.978$
- Fitting data generated by FactSage software.

Source: XU, J. et alii. *Effects of Elements on Peritectic Reaction in Molten Steel Based on Thermodynamic Analysis*. **ISIJ International**, 52:10, October 2012, 1856-1861.

- **Cooling Rate of Flat Products of Steel**

. Bodnar

$$\Delta t_{8/5} = 946\ t^{-1.032}$$

Notation:

$\Delta t_{8/5}$: Average Cooling Rate of Steel Plate in Air Between 800°C and 500°C [°C/min]
t: Plate Thickness [mm]

Observation:

- Temperature evolution was measured at plate quarter-thickness.

Source: BODNAR, R.L. et alii: *The Physical Metallurgy of Normalized Plate Steels*. In: **Materials Science & Technology**, AIST, New Orleans, vol. 1, 2004, 89-109.

. Degenkolbe

$$CR_{8/5} = 4.52762\ t^{-0.74694}$$

Notation:

$CR_{8/5}$: Cooling Rate of in the Core of a Steel Plate in Air Between 800°C and 500°C [°C/s]
t: Plate Thickness [mm]

Observation:

- Equation fitted by the author of this review using data available in the figure 2 of the reference below.

Source: DEGENKOLBE, J. et alii: *Erfahrungen mit des beschleunigten Abkühlung von Grobblech aus der Walthitze*. **Thyssen Technische Berichte,** 1, 1987, 41-55.

- **Density of Bulk Steel at Ambient Temperature**

. Austenitic Steels

$$\rho_\gamma = \frac{1}{(1.231\,Fe + 3.178\,C_{sol} + 1.307\,Mn + 2.436\,Si + 1.431\,Cr + 1.205\,Cu + 1.018\,Mo + 1.137\,Ni + 1.890\,Ti + 1.111\,Co + 2.186\,N + 2.032\,TiC).10^{-6}}$$

Notation:

ρ_γ: Austenite Density [kg/m³]
Alloy/TiC Content: [weight %]

Observations:

- C_{sol} is the content of this element not bound in TiC.
- Density calculated at 20°C.

Source: BOHNENKAMP, U. et alii.: *Evaluation of the Density of Steels*. **Steel Research**, 71:3, March 2000, 88-93.

. Ferritic Steels

$$\rho_\alpha = \frac{1}{(1.270\,Fe + 1.380\,C + 1.524\,Mn + 2.381\,Si + 1.384\,Cr + 0.8477\,Cu + 1.076\,Mo + 1.370\,Ni + 2.012\,V + 4.046\,S).10^{-6}}$$

Notation:

ρ_α: Ferrite Density [kg/m³]
Alloy Content: [weight %]

Observations:

- C is considered insoluble in ferrite (that is, all C has gone to cementite).
- The solubilities of the other alloy elements in cementite are zero.
- Density calculated at 20°C.

Source: BOHNENKAMP, U. et alii.: *Evaluation of the Density of Steels*. **Steel Research**, 71:3, March 2000, 88-93.

. Density of Fe-C Alloys in Heterogeneous Phase Mixtures

$$\frac{1}{\rho_{Steel}} = \frac{f_1}{\rho_1} + \frac{f_2}{\rho_2} + \frac{f_3}{\rho_3} + ... + \frac{f_n}{\rho_n}$$

Notation:

ρ_{Steel}: Steel Density [kg/m³]
f_i: Fraction of the phase **i** in the microstructure
ρ_i: Density of the phase **i**

Source: JABLONKA, A.: *Thermomechanical Properties of Iron and Iron-Carbon Alloys: Density and Thermal Contraction,* **Steel Research**, 62:1, September 1991, 24-33.

- **Density of Bulk Steel at High Temperature**

. BISRA

T	ρ			
	1008	**1023**	**1040**	**1524**
0	7861	7863	7858	7854
15	7856	7859	7854	7849
50	7847	7849	7845	7840
100	7832	7834	7832	7826
150	7816	7819	7817	7811
200	7800	7803	7801	7794
250	7783	7787	7784	7777
300	7765	7770	7766	7760
350	7748	7753	7748	7742
400	7730	7736	7730	7723
450	7711	7718	7711	7704
500	7792	7699	7692	7685
550	7673	7679	7672	7666
600	7653	7659	7652	7646
650	7632	7635	7628	7622
700	7613	7617	7613	7605
750	7594	7620	7624	7615
800	7582	7624	7643	7641
850	7589	7625	7617	7614
900	7600	7600	7590	7590

950	7572	7574	7564	7561
1000	7543	7548	7538	7532
1050	7515	7522	7512	7503
1100	7488	7496	7486	7474

Notation:

ρ: Density of steel [kg/m³]
T: Temperature [°C]

Observations:

- Chemical composition of the steels [wt %]:

Steel	**C**	**Mn**	**Si**	**P**	**S**	**Cu**
1008	0.08	0.31	0.08	0.029	0.050	-
1023	0.23	0.64	0.11	0.034	0.034	0.13
1040	0.42	0.64	0.11	0.031	0.029	0.12
1524	0.23	1.51	0.12	0.037	0.038	0,11

Source: *Physical Constants of Some Commercial Steels at Elevated Temperatures*, BISRA/Butterworths Scientific Publications, London, 1953, 1-38.

. Picquè

$$\rho = 7875.96 - 0.297\,T - 5.62 \times 10^{-5}\,T^2 \quad (T \le Ar_3)$$

$$\rho = 8099.79 + 0.506\,T \quad (T > Ar_3)$$

Notation:

ρ: Density of steel [kg/m³]
T: Temperature [°C]

Observation:
- Formulas specific for a 0.16% C, 0.5% Mn steel

Source: PICQUÉ, B. *Experimental Study and Numerical Simulation of Iron Oxide Scales Behavior in Hot Rolling*. **Doctor Thesis**, École de Mines de Paris, 2004, p. 247.

- **Density of Liquid Steel**

. Jablonka

$$\rho_{Steel} = (8319.49 - 0.835\,T)\,(1 - 0.01\,C)$$

Notation:

ρLiquid Iron: Liquid Iron Density [kg/m³]
T: Temperature, [°C]
C: Carbon content [weight %]

Source: JABLONKA, A.: *Thermomechanical Properties of Iron and Iron-Carbon Alloys: Density and Thermal Contraction,* **Steel Research**, 62:1, September 1991, 24-33.

. Yaws

$$\rho_{Liquid\ Iron} = 1.9946 \times 0.22457^{-\left(1 - \frac{T}{9340}\right)^{0.7}}$$

Notation:

ρLiquid Iron: Liquid Iron Density [g/ml]
T: Absolute Temperature, [K]

Source: YAWS, C.L.: *Liquid Density of the Elements*. **Chemical Engineering**, November 2007, 44-46.

- **Density of Microstructural Constituents at Ambient Temperature**

. Common Phases and Constituents

Phase/Constituent	C [weight %]	Specific Volume [cm³/g] at 20°C
Austenite	0.00 ~ 2.00	0.1212 + 0.0033 C
Martensite	0.00 ~ 2.00	0.1271 + 0.0025 C
Ferrite	0.00 ~ 0.02	0.1271
Cementite (Fe_3C)	6.7 ± 0.2	0.130 ± 0.001
ε Carbide	8.5 ± 0.7	0.140 ± 0.002
Graphite	100	0.451
Ferrite + Cementite	0.00 ~ 2.00	0.1271 + 0.0005 C
Low C Martensite + ε Carbide	0.25 ~ 2.00	0.1277 + 0.0015 (C – 0.25)
Ferrite + ε Carbide	0.00 ~ 2.00	0.1271 + 0.0015 C

Notation:

C: Carbon Content [weight %]

Source: THELNING, K.E.: *Steel and its Heat Treatment – Bofors Handbook*. Butterworths, London, 1981, 570 p.

. Density and Molar Volume of Microalloy Carbides and Nitrides

Compound	Structure	Molecular Mass	Lattice Parameter [nm]	Molecules per Unit Cell	Density [g/cm³]	Molar Volume [cm³/mol]
NbC	FCC	105	0.4462	4	7.84	13.39
NbN	FCC	107	0.4387	4	8.41	12.72
VC	FCC	63	0.4154	4	5.83	10.81
VN	FCC	65	0.4118	4	6.18	10.52

TiC	FCC	60	0.4313	4	4.89	12.27
TiN	FCC	62	0.4233	4	5.42	11.44
AlN	CPH	41	c = 0.4965 a = 0.311	6	3.27	12.54
γ-Fe	FCC	56	0.357	4	8.15	6.85
α-Fe	BCC	56	0.286	2	7.85	7.11

Observations:

- Data based on room temperature lattice parameters.

Source: GLADMAN, T. *The Physical Metallurgy of Microalloyed Steels*. The Institute of Materials, London, 1997, 363 p.

. Density and Molar Volume of Precipitates and Metals

Compound	**Density [kg/m³]**	**Molar Volume [cm³/mol]**
NbCN	9291	12,80
ZrC	6,572	-
ZrN	7,30	-
Mn	7470	7,35
Si	2330	12,06
Cr	7140	7,23
Cu	8920	7,11
C	2267	5,29
N	-	13,54
Steel	7850	7,00

Observations:

- Data based on room temperature lattice parameters.

Sources:

- SAN MARTIN, D. et alii. *Estudio y Modelización de la Influencia de las Partículas de Segunda Fase sobre el Crecimiento de Grano Austenítico en um Acero Microaleado com Niobio*. **Revista de Metalurgia – Madrid**, 42:2, Marzo-Abril 2006, 128-137.

- Web Elements (www.webelements.com)

- NISHIZAWA, T. *Thermodynamics of Microstructure Control by Particle Dispersion*. **ISIJ International**, 40:12, December 2000, 1269-1274.

- ADRIAN, H. *Thermodynamical Model for Precipitation of Carbonitrides in HSLA Steels Containing Up to Three Microalloying Elements with or without Additions of Aluminum*. **Materials Science and Tecnology**, 8:5, May 1992, 406-420.

. Relationship Between Lattice Parameter and Density

$$\rho = \frac{n\,M}{(a\ 10^{-10})^3\ N}$$

Notation:

ρ: Density [kg/m³]
n: Number of atoms per unit cell (depends on crystalline structure):
. Cubic body-centered: 2
. Cubic face-centered: 4
M: Molecular mass [kg/mol]:
. Pure Fe: 0.055847
. Cementite: 0.179552
a: Lattice Parameter [Å]
N: Avogrado's number: $6.023 \cdot 10^{23}$

Source: JABLONKA, A.: *Thermomechanical Properties of Iron and Iron-Carbon Alloys: Density and Thermal Contraction,* **Steel Research**, 62:1, September 1991, 24-33.

- **Density of Microstructural Constituents at High Temperature**

. Fink

$$\rho_{\gamma}^{T} = \rho_{\gamma}^{20°C} - 0.47\,T$$

$$\rho_{\alpha}^{T} = \rho_{\alpha}^{20°C} - 0.33\,T$$

Notation:

ρ_{γ}^{T}: Austenite Density at Temperature T [kg/m³]
$\rho_{\gamma}^{20°C}$: Austenite Density at 20°C [kg/m³]
ρ_{α}^{T}: Ferrite Density at Temperature T [kg/m³]
$\rho_{\alpha}^{20°C}$: Ferrite Density at 20°C [kg/m³]
T: Temperature[°C]

Source: FINK, K. et alii.: *Physikalische Eigenschaften von Stählen, insbesondere von warmfesten Stählen.* **Thyssenforschung**, 2:2, 1970, 65-80.

. Jablonka

$$\rho_{\partial}^{T} = \left(7875.96 - 0.297\,T - 5.62\times10^{-5}\,T^{2}\right)\left(1 - 2.62\times10^{-2}\,C\right)$$

$$\rho_{\gamma}^{T} = \left(8099.79 - 0.506\,T\right)\left(1 - 1.46\times10^{-2}\,C\right)$$

$$\rho_{\alpha}^{T} = \left(7875.96 - 0.297\,T - 5.62\times10^{-5}\,T^{2}\right)\left(1 - 2.62\times10^{-2}\,C\right)$$

$$\rho_{Fe3C}^{T} = \left(7686.45 - 6.63\times10^{-2}\,T - 3.12\times10^{-4}\,T^{2}\right)$$

Notation:

ρ_δ^T: Delta Ferrite Density at Temperature T [kg/m³]
ρ_γ^T: Austenite Density at Temperature T [kg/m³]
ρ_α^T: Ferrite Density at Temperature T [kg/m³]
ρ_{Fe3C}^T: Cementite Density at Temperature T [kg/m³]
T: Temperature[°C]
C: Carbon content [weight %]

Observations:

- Carbon content in ferrite is limited to 0.02% maximum.

Source: JABLONKA, A.: *Thermomechanical Properties of Iron and Iron-Carbon Alloys: Density and Thermal Contraction,* **Steel Research**, 62:1, September 1991, 24-33.

. Molar Volume of Austenite as Function of Temperature

$$V_M^\gamma = 6.688726 \times 10^{-6} \exp\left(7.3097 \times 10^{-5}\ T\right)$$

Notation:

V_M^γ: Molar Volume of Austenite, [m³/mol]
T: Temperature [K]

Source: FERNÁNDEZ, D.M.S.M. *Modelización de la Cinética de Austenización y Crecimiento de Grano Austenítico en Aceros Ferrítico-Perlíticos*. **Tesis Doctoral**, Centro Nacional de Investigaciones Metalúrgicas, Madrid, Julio 2003, 258 p.

. Relationship Between Density and Thermal Expansion

$$\varepsilon^{th} = \sqrt[3]{\frac{\rho(T_0)}{\rho(T)}} - 1$$

Notation:

ε^{th}: Thermal Expansion/Contraction
$\rho(T_0)$: Density at lower/higher T_0
$\rho(T)$: Densotu at temperature T
T_0: Reference temperature

Source: JABLONKA, A.: *Thermomechanical Properties of Iron and Iron-Carbon Alloys: Density and Thermal Contraction,* **Steel Research**, 62:1, September 1991, 24-33.

- **Dimensional Changes during Austenite Transformation**

. During Cooling

$$\frac{\Delta L_{\gamma\to\alpha}}{L} = 1.232\times10^{-2} - 1.347\times10^{-5}\ T - 6.544\times10^{-3}\ C_\gamma - 5.829\times10^{-3}\ C_\gamma^2 + 9.766\times10^{-6}\ C_\gamma\ T + 2.379\times10^{-9}\ T^2$$

$$C_\gamma = \frac{C_0\ (1 - X_A)\ C_{tr}}{X_A}$$

Notation:

$\Delta L_{\gamma\to\alpha}$: Specimen Length Change During Austenite Transformation [µm];
L: Specimen Original Length [µm]
T: Temperature [°C]
C_γ: Carbon Content in Austenite [wt %]
C_0: Initial Carbon Content [wt %]
C_{tr}: Carbon Content of Transformed Phases [wt %]
X_A: Untransformed Austenite Volume Fraction

Source: PARK, S.H. *Microstructural Evolution of Hot Rolled TRIP Steels During Cooling Control*. In: **40th Mechanical Working and Steel Processing Conference**, ISS/AIME, Pittsburgh, October 1998, 283-291.

. After General Heat Treating

Transformation	**ΔV [%]**	**Δl [mm/mm]**

Spheroidized Pearlite → Austenite	-4.64 + 2.21 C	-0.0155 + 0.0074 C
Austenite → Martensite	4.64 – 0.53 C	0.0155 – 0.0018 C
Spheroidized Pearlite → Martensite	1.68 C	0.0056 C
Austenite → Lower Bainite	4.64 – 1.43 C	0.0155 – 0.0048 C
Spheroidized Pearlite → Lower Bainite	0.78 C	0.0026 C
Austenite → Upper Bainite	4.14 – 2.21 C	0.0155 – 0.0074 C
Spheroidized Pearlite → Upper Bainite	0 (Zero)	0

Notation:

- C: Carbon Content [weight %].

Sources:

- THELNING, K.E.: *Steel and its Heat Treatment – Bofors Handbook*. Butterworths, London, 1981, 570 p.

- KRAUSS, G. *Steel: Processing, Structure and Performance*. ASM International, Metals Park, 2005, 420 p.

. After Quenching

$$\frac{\Delta V}{V} = \left(\frac{100 - V_C - V_A}{100} \right) 1.68\, C_M + \frac{V_A}{100} (-4.64 + 2.21\, C_A)$$

Notation:

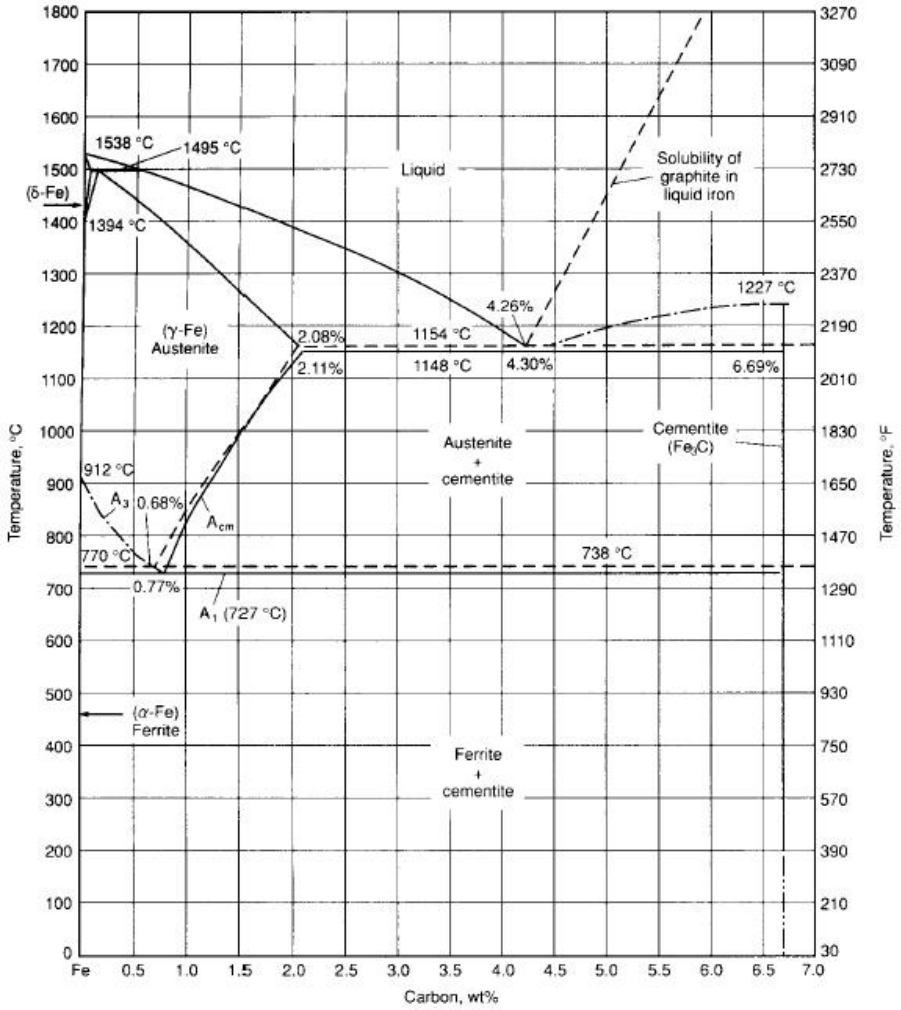
- $\Delta V/V$: Volumetric Change after Quenching [%]
- V_C: Non-solubilized Cementite Volumetric Fraction [%]
- V_A: Austenite Volumetric Fraction [%]
- $100 – V_C – V_A$: Martensite Volumetric Fraction [%]
- C_M: Carbon Content Solubilized in Martensite [weight %]

- C_A: Carbon Content Solubilized in Austenite [weight %]

Source: THELNING, K.E.: *Steel and its Heat Treatment – Bofors Handbook*. Butterworths, London, 1981, 570 p.

-
-

- **Fe-C Equilibrium Diagram**



Source: **ASM's Heat Treating One-Minute Mentor**,
http://www.asminternational.org/pdf/HTSRefCharts/Vol4p4Fig1.pdf.

- **Fe-C Equilibrium Equations in the Solidification and Eutectoid Range**

. Liquidus of δ-iron

$$\%C = \frac{1536 - T}{79}$$

. Liquidus of γ-iron

$$\%C = \frac{1525 - T}{56.0198} - \frac{1.1284 \times 10^{-3}\ (T - 1525)^2}{56.0198}$$

Notation:

%C: coordinate in the Fe-C diagram [%]
T: Temperature [°C]

. Solidus of δ-iron

$$\%C = \frac{1536 - T}{460}$$

. Solidus of γ-iron

$$\%C = \frac{1525 - T}{185}$$

. Start of transformation of δ-iron (on cooling)

$$\%C = \frac{T - 1392}{1140}$$

. End of transformation of δ-iron (on cooling)

$$\%C = \frac{T - 1392}{624.3749}$$

. A_3 Line

$$\%C = \frac{911 - T}{484.785} + \frac{2.818 \times 10^{-4}\ (T - 911)^2}{484.785} - \frac{2.8574 \times 10^{-5}\ (T - 911)^3}{484.785}$$

. A_{cm} Line

$$\%C = 0.8 + \frac{T - 723}{453.7137} + \frac{7.7917 \times 10^{-4}\ (T - 723)^2}{453.7137}$$

Source: JABLONKA, A.: *Thermomechanical Properties of Iron and Iron-Carbon Alloys: Density and Thermal Contraction,* **Steel Research**, 62:1, September 1991, 24-33.

- **Ferrite Solubility Products**

. General

$$\log_{10} \frac{(a_A)^m \, (a_B)^n}{a_{A_m B_n}} = -\frac{A}{T} + B$$

Notation:

A_mB_n: Precipitate Considered for Calculation
a_x: Alloy Content [weight %]
T: Temperature [K]
A, B: Constants of the Solubility Product, given in the table below:

Precipitate	A	B	Source
AlN	9595	2.65	Kunze & Reichert
BN	13560	4.53	Fountain & Chipman
MnS	8400	2.77	Ivanov
NbC	10990	4.62	Kunze
NbN	10650	3.87	Kunze
TiN	17640	6.17	Kunze
VC	12265	8.05	Taylor
VN	7830	2.45	Froberg
VN	8120	2.48	Roberts & Sandbert
ZrN	18160	5.24	Kunze

Observations:

- a_{AmBn} is equal to one if the precipitate is pure.
- $a_{AmBn} \leq 1$ if there is co-precipitation with another element.

Sources:

- TAYLOR, K.A. et alii. **Scripta Metallurgica**, 32, 1995, 7.

- FROBERG, M.G. & GRAF, H. **Stahl und Eisen**, 80, 1960, 539.

- KUNZE, J. **Nitrogen and Carbon in Iron and Steels – Thermodynamics.** Akademie Verlag, Berlin, 1991, p. 192.

- FOUNTAIN, R.W. & CHIPMAN, J. In: **Transactions of the Metallurgical Society of AIME**, 224, 1964, 599.

- KUNZE, J. & REICHERT, J. **Neue Hütte**, 26, 1981, 23.

- ROBERTS, W. & SANDBERG, A. **Report IM 1489**. Institute for Metallurgical Research, Stockholm, 1990.

- IVANOV, B.S. et alii. **Stahl**, 8, 1996, 52.

- **Hardness After Austenite Cooling**

. Blondeau

$$HV = f_{FP}\,HV_{FP} + f_B\,HV_B + f_M\,HV_M$$

$$HV_{FP} = 42 + 223\,C + 53\,Si + 30\,Mn + 7\,Cr + 19\,Mo + 126\,Ni + (10 - 19\,Si + 8\,Cr + 4\,Ni + 130\,V)\log_{10}(v_r)$$

$$HV_B = -323 + 185\,C + 330\,Si + 153\,Mn + 144\,Cr + 191\,Mo + 65\,Ni + (89 + 53\,C - 55\,Si - 22\,Mn - 20\,Cr - 33\,Mo - 10\,N)\log_{10}(v_r)$$

$$HV_M = 127 + 949\,C + 27\,Si + 11\,Mn + 16\,Cr + 8\,Ni + 21\log_{10}(v_r)$$

$$\log(v_M) = 9.81\,(-4.62\,C + 1.05\,Mn + 0.54\,Ni + 0.50\,Cr + 0.66\,Mo + 0.00183\,PA)$$

$$\log(v_{M90}) = 8.76 - (4.04\,C + 0.96\,Mn + 0.58\,Cr + 0.97\,Mo + 0.49\,Ni + 0.001\,PA)$$

$$\log(v_{M50}) = 8.50 - (4.13\,C + 0.86\,Mn + 0.41\,Cr + 0.94\,Mo + 0.57\,Ni + 0.0012\,PA)$$

$$\log(v_B) = 10.17 - (3.80\,C + 1.07\,Mn + 0.70\,Ni + 0.57\,Cr + 1.58\,Mo + 0.0032\,PA)$$

$$\log(v_{B90}) = 10.55 - (3.65\,C + 1.08\,Mn + 0.61\,Cr + 1.49\,Mo + 0.77\,Ni + 0.0032\,PA)$$

$$\log(v_{B50}) = 8.74 - (2.23\,C + 0.86\,Mn + 0.59\,Cr + 1.60\,Mo + 0.56\,Ni + 0.032\,PA)$$

$$\log(v_{FP}) = 6.36 - (0.43\,C + 0.49\,Mn + 0.78\,Ni + 0.27\,Cr + 0.38\,Mo + 2\sqrt{Mo} + 0.0019\,PA)$$

$$\log(v_{FP90}) = 7.51 - (1.38\,C + 0.35\,Mn + 0.11\,Cr + 2.31\,Mo + 0.93\,Ni + 0.0033\,PA)$$

$$PA = \left[\frac{1}{T} - \frac{4.58\, log_{10}(t)}{\Delta H}\right]^{-1}$$

Notation:

HV: Global Hardness [Vickers]
f_{FP}: Fraction of Ferrite-Pearlite in Microstructure
f_B: Fraction of Bainite in Microstructure
f_M: Fraction of Martensite in Microstructure
HV_{FP}: Hardness of Ferrite-Pearlite [Vickers]
HV_B: Hardness of Bainite [Vickers]
HV_M: Hardness of Martensite [Vickers]
Alloy Content: [weight %]
v_r: Applied Cooling Rate at 700°C [°C/h]
v_M: Critical Cooling Rate at 700°C for Martensitic Quenching [°C/h]
v_{M90}: Cooling Rate at 700°C Required for 90% Martensite [°C/h]
v_{M50}: Cooling Rate at 700°C Required for 50% Martensite [°C/h]
v_B: Critical Cooling Rate at 700°C for Bainitic Quenching [°C/h]
v_{B90}: Cooling Rate at 700°C Required for 90% Bainite [°C/h]
v_{B50}: Cooling Rate at 700°C Required for 50% Bainite [°C/h]
v_{FP}: Critical Cooling Rate at 700°C for Annealing (Ferrite-Pearlite) [°C/h]
v_{FP90}: Cooling Rate at 700°C Required for 90% Ferrite-Pearlite [°C/h]
PA: Austenitization Parameter [K]
T: Austenitization Temperature [K]
t: Austenitization Soaking Time [h]
ΔH: Austenitization Activation Energy: 240 kJ/mol for C Steels; 418 kJ/mol if Mo ≥ 0.04%, 461 kJ/mol for most alloy steels

Observations:

- Limits for Austenitization: 1073 K (800°C) ≤ T ≤ 1373 K (1100°C) and t ≤ 1 h.
- Equations Valid for the Following Chemical Composition Range: 0.10% < C < 0.50%, Mn < 2.0%, Si < 1.0%, Ni ≤ 4.0%, Cr < 3.0%, Mo < 1.0%, Cu < 0.5%, V < 0.2%, 0.010% < Al < 0.050% and Mn + Ni + Cr + Mo < 5.0%.

Source: BLONDEAU, R. et alii.: *Mathematical Model for the Calculation of Mechanical Properties of Low-Alloy Steel Products: A Few Examples of its Application*. In: **16th International Heat Treatment Conference – Heat Treating '76**, The Metals Society Stratford-upon-Avon, 1976, 189-200.

. Lorenz

$$HV = 2019\left[C\,(1-0.5\log t_{8/5}) + 0.3\left(\frac{Si}{11}+\frac{Mn}{8}+\frac{Cu}{9}+\frac{Cr}{5}+\frac{Ni}{17}+\frac{Mo}{6}+\frac{V}{3}\right)\right] + 66\,(1-0.8\log t_{8/5})$$

Notation:

HV: Maximum Hardness for a Martensitic-Bainitic HAZ Microstructure [Vickers, 10 kg Load]
Alloy Content: [weight %]
$t_{8/5}$: Cooling Time Between 800°C and 500°C [s]

Source: LORENZ, K. et alii.: *Evaluation of Large Diameter Pipe Steel Weldability by Means of the Carbon Equivalent.* In: **International Conference on Steels for Linepipe and Fittings**, Metals Society, London, Oct. 1981, 322-332.

. Lotter

$\Delta t_{8/5}$ = 2 s

$$HV_{10} = 177.7 + 975.4\,C + 42.4\,Mn + 163.3\,Mo - 1247\,Nb$$

- r = 0.89; Root Mean Square Deviation = 54 HV_{10}

$\Delta t_{8/5}$ = 10 s

$$HV_{10} = 103.6 + 1094.5\,C + 346.0\,Mo$$

- r = 0.91; Root Mean Square Deviation = 49 HV_{10}

$\mathbf{\Delta t_{8/5}}$ = 50 s

$$HV_{10} = 86.8 + 341.5\,C + 52.0\,Mn - 85.9\,Si + 152.2\,Cr + 275.9\,Mo$$

- r = 0.88; Root Mean Square Deviation = 37 HV_{10}

$\mathbf{\Delta t_{8/5}}$ = 100 s

$$HV_{10} = 79.5 + 317.5\,C + 57.4\,Mn - 107.1\,Si + 141.0\,Cr + 225.9\,Mo$$

- r = 0.88; Root Mean Square Deviation = 33 HV_{10}

$\mathbf{\Delta t_{8/5}}$ = 500 s

$$HV_{10} = 61.7 + 277.9\,C + 34.9\,Mn + 94.7\,Cr + 96.7\,Cu + 165.1\,Mo$$

- r = 0.89; Root Mean Square Deviation = 27 HV_{10}

$\mathbf{\Delta t_{8/5}}$ = 1000 s

$$HV_{10} = 56.6 + 262.6\,C + 33.7\,Mn + 74.9\,Cr + 121.8\,Cu + 155.9\,Mo$$

- r = 0.88; Root Mean Square Deviation = 26 HV_{10}

Observations:

- These formulas were determined using data got from samples submitted to a normalizing rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; final cooling.

$\mathbf{\Delta t_{8/5}}$ = 2 s

$$HV_{10} = 107.3 + 1107.5\,C + 47.4\,Mn + 60.9\,Ni + 204.9\,Mo - 22856.0\,B$$

- r = 0.97; Root Mean Square Deviation = 32 HV_{10}

$\mathbf{\Delta t_{8/5}}$ = 10 s

$$HV_{10} = 84.2 + 773.5\,C + 43.6\,Mn + 106.5\,Cr + 192.3\,Mo + 28510.0\,B$$

- r = 0.92; Root Mean Square Deviation = 40 HV_{10}

$\mathbf{\Delta t_{8/5}}$ = 50 s

$$HV_{10} = 36.6 + 398.0\,C + 81.6\,Mn - 138.1\,Si + 191.7\,Cr + 246.7\,Mo + 302.0\,Ti + 1018.0\,Nb$$

- r = 0.95; Root Mean Square Deviation = 23 HV_{10}

$\mathbf{\Delta t_{8/5}}$ = 100 s

$$HV_{10} = 25.4 + 371.2\,C + 77.7\,Mn - 103.9\,Si + 186.8\,Cr + 216.7\,Mo + 241.0\,Ti + 1095.0\,Nb$$

- r = 0.96; Root Mean Square Deviation = 19 HV_{10}

$\mathbf{\Delta t_{8/5}}$ = 500 s

$$HV_{10} = 25.7 + 318.8\,C + 61.0\,Mn - 60.4\,Si + 135.3\,Cr + 70.1\,Cu + 201.9\,Mo + 224.0\,Ti + 853.0\,Nb$$

- r = 0.95; Root Mean Square Deviation = 18 HV_{10}

$\mathbf{\Delta t_{8/5}}$ = 1000 s

$$HV_{10} = 37.1 + 282.2\,C + 44.1\,Mn - 60.9\,Si + 113.2\,Cr + 173.4\,Cu + 124.7\,Mo + 420.0\,V + 236.0\,Ti + 550.0\,Nb$$

- r = 0.98; Root Mean Square Deviation = 13 HV_{10}

Observations:

- These formulas were determined using data got from samples submitted to a thermomechanical rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; 35% @800°C; final cooling.

Notation:

$\mathbf{\Delta t_{8/5}}$: Time Between 800°C and 500°C [s]
$\mathbf{HV_{10}}$: Vickers Hardness, 10 kg Load]
Alloy Amount: [weight %]

Source: LOTTER, U. *Aufstellung von Regressionsgleichungen zur Beschreibung des Umwandlungsverhaltens beim thermomechanischen Walzen.* Technische Bericht, Thyssen Stahl, Duisburg, 1988, 136 p.

. Murry

$$HV = HV_{\max} - \left(HV_{\max} - HV_0\right)\exp\left(-\frac{0.6\,U\,\Delta t_0^{\,n}}{\Delta t^{\,n} - \Delta t_0^{\,n}}\right)$$

$$\Delta t_0 = C\exp\left[2.3\left(\alpha_0 - \frac{0.04}{\sqrt{d}} + \sum \alpha_i\,C_i\right)\right]$$

$$n = C \exp\left[2.3\left(\beta_0 + \frac{0.03}{\sqrt{d}} + \frac{0.09}{C} - \sum \beta_i \, C_i^{0.43}\right)\right]$$

$$U = 1 + p \exp\left[-0.35\left(\ln \frac{\Delta t}{\Delta t_M}\right)^2\right]$$

$$\Delta t_M = \Delta t_0 \, 12^{\frac{1}{n}}$$

$$p = \left(\frac{C}{\Delta t_0}\right)^{3.1} + \exp\left[2.3\left(\gamma_0 + \sum \gamma_i \, C_i\right)\right]$$

$$HV_0 = HV_{\min} + 0.055\left(HV_{\max} - 300\right)\left(9 - \log \Delta t_{300}^{700}\right)$$

$$HV_{\min} = 63 + \sum_i \mu_i \, Z_i^{0.75}$$

Notation:

HV_{max}: Maximum Hardness for a Martensitic Structure
C: Carbon [weight %]
C_i: Alloy Content [weight %]
d: Mean Austenite Grain Size [mm]
Δt^{700}_{300}: Cooling Time Between 700°C and 300°C [s]
HV_{min}: Minimum Hardness at Equilibrium Calculated According to the Substitution Solid Solution Hardening Effect Proposed by Lacy and Gensamer
Z_i: Concentration of the Element **i** in solid solution with ferrite at equilibrium [at %]
μ_i: Action Coefficient of the Element **i**, as described in the following table:

Element	Mn	Si	P	Ni	Cr	Mo	V	W
μ_i	14.27	22.42	61.16	12.44	2.84	19.57	8.15	22.42

General Constants: α_0 = 0.89, β_0 = 1.22, γ_0 = 1.82
Constants: α_i, β_i and γ_i according to the alloy element:

Element i	α_i	β_i	γ_i
Mn	0.39	0.94	1.40
Si	0.20	0.15	0.80
Ni	0.22	0.40	0.12
Cr	067	0.09	2.40
Mo	0.17	0.72	0.79
V	0.20	0.50	0.90
Nb	0.40	1.20	1.00

Observations:

- Equations Valid Within the Following Chemical Composition Range: 0.05% ≤ C ≤ 0.80%, 0.50% ≤ Mn ≤ 2.5%, 0.15% ≤ Si ≤ 0.35%, Ni ≤ 1.0%, Cr ≤ 1.5%, Mo ≤ 0.5%, V ≤ 0.1%, Nb ≤ 0.040%.

Source: MURRY, G.: **Transformations Dans les Aciers**, Document M 1 115, Techniques de l'Ingénieur, Paris, 1985, 54 p.

. Trzaska

. Specific equations:

$$HV_{\alpha p} = -73 + 253\,C + 52\,Mn + 10\,Si + 36\,Cr + 8\,Ni + 20\,Mo + 80\,V + 0.11\,T_A + 12.5\sqrt[4]{v_c}$$

$$HV_m = 200 + 824\,C + 44\,Mn + 14\,Cr + 9\,Ni + 171\,V + 78.5\,Cu + 4.13\sqrt[4]{v_c}$$

. General equation:

$$HV = 3.7 + 225\,C + 82\,Mn + 28\,Si + 55\,Cr + 28\,Ni + 53.5\,Mo + 147\,V + 71\,Cu + 0.09\,T_A - 3.8\sqrt[4]{v_c} + 68\,C\sqrt[4]{v_c} - 42\,W_f - 69\,W_p - \\ - 32.5\,W_b + 72\,W_m$$

$$W_x = \begin{cases} 0\ if\ S_X \le N \\ 1\ if\ S_X > N \end{cases}$$

$$S_x = \frac{e^{K_x}}{1 + e^{K_x}}$$

$$K_f = 18.4 - 15.4\,C - 1.9\,Mn + 0.7\,Si - 2.5\,Cr - 1.5\,Ni - 4.8\,Mo + 2.4\,V + 1.4\,Cu - 0.004\,T_A + \sqrt[4]{v_c}$$

$$K_p = 12 - 1.4\,C - 2.3\,Mn - 2.3\,Cr - 1.4\,Ni - 6\,Mo + 3.9\,V - 0.002\,T_A - 1.2\sqrt[4]{v_c}$$

$$K_b = 1.3 - 3.7\,C + 0.45\,Mn + 0.2\,Cr + 0.18\,Ni + 1.9\,Mo - 0.17\sqrt[4]{v_c} - 0.57\sqrt{(4.35 - \sqrt[4]{v_c})^2}$$

$$K_m = -16.5 + 4.7\,C + 2.6\,Mn + 0.6\,Si + 2.4\,Cr + 1.2\,Ni + 1.9\,Mo + 4.8\,Cu + 0.006\,T_A + 1.1\sqrt[4]{v_c}$$

Notation:

HV_{ap}: Ferrite-Pearlite Hardness [Vickers]
HV_m: Martensite Hardness [Vickers]

HV: General Hardness [Vickers]
Alloy Content: [weight %]
$\mathbf{T_A}$: Austenitizing Temperature [°C]
$\mathbf{v_c}$: Cooling Rate [°C/min]
x: f (ferrite), p (pearlite), b (bainite) or m (martensite)
N: 0.5 for ferrite (f), pearlite (p) and martensite (m); 0.4 for bainite (b).

Observations:

- Formula valid within the following range: 0.06% ≤ C ≤ 0.68%, 0.13% ≤ Mn ≤ 2.04%, 0.12% ≤ Si ≤ 1.75%, Cr ≤ 2.30%, Ni ≤ 3.85%, Mo ≤ 1.05%, V ≤ 0.38% and Cu ≤ 0.38%.
- Additional conditions: Mn+Cr ≤ 3.6%. Mn+Cr+Ni ≤ 5.6%, Cr+Ni ≤ 5.3%, Mn+Ni ≤ 4.5%.
- HV: $r^2 = 0.847$, standard error = 62.3 HV, mean absolute error = 48.5 HV.
- $HV_{\alpha p}$: $r^2 = 0.743$, standard error = 24.4 HV, mean absolute error = 19.4 HV.
- HV_m: $r^2 = 0.855$, standard error = 39.9 HV, mean absolute error = 30.5 HV.

Source: TRZASKA, J.: *Empirical Formulae for the Calculation of the Hardness of Steels Cooled from the Austenitizing Temperature*. **Archives of Metallurgy and Materials**, 61:3, 2016, 1297-1302.

- **Hardness After Tempering**

. Spies

$$HB = 2.84\,HRC + 75\,C - 0.78\,Si + 14.24\,Mn + 14.77\,Cr + 128.22\,Mo - 54.0\,V - 0.55\,T + 435.66$$

Notation:

HB: Brinell Hardness After Hardening and Tempering
HRC: Rockwell Hardness (C Scale) After Hardening
Alloy Content: [weight %]
T: Tempering Temperature [°C]

Observations:

- This equation is valid within the following ranges: **HRC:** 20~65; **C:** 0.20~0.54%; **Mn:** 0.50~1,90%; **Si:** 0.17~1.40%; **Cr:** 0.03~1.20%; **T:** 500~650°C.

Source: SPIES, H.J. et alii.: *Möglichkeiten der Optimierung der Auswahl vergütbarer Baustähle durch Berechnung der Härt-und-vergütbarkeit.* **Neue Hütte**, 8:22, 1977, 443-445.

- **Hardness After Welding**

. Dearden & O'Neill

$$HV_{\max} = 1200\ C_{EQ_Dearden} - 200$$

$$C_{EQ_Dearden} = C + \frac{Mn}{6} + \frac{Mo}{4} + \frac{Cr+V}{5} + \frac{Cu}{13} + \frac{Ni}{15} + \frac{P}{2}$$

Notation:

$C_{EQ_Dearden}$: Equivalent Carbon (Dearden) [%]
Alloy Content: [weight %]
HV_{max} = Maximum Hardness [Vickers]

Observations:

- This equation calculates maximum hardness after welding.

Source: DEARDEN, J & O'NEIL, H.: *A Guide to the Selection and Welding of Low Alloy Structural Steels*. **Transactions of the Institute of Welding**, 3, 1940, 203-214.

. Khan

$$HV = 188 + 630\ C_{EQ_Yurioka}$$

Notation:

$C_{EQ_Yurioka}$: Equivalent Carbon for Pipeline Steels [%]

$$C_{EQ_Yurioka} = C + A(C)\left(\frac{Mn}{6} + \frac{Si}{24} + \frac{Cr + Mo + V}{5} + \frac{Cu}{15} + \frac{Ni}{20} + \frac{Nb}{5} + 5\ B\right)$$

$$A(C) = 0.75 + 0.25 \tanh\left[20\,(C - 0.12)\right]$$

Alloy Content: [weight %]

Observations:

- HV is the fusion zone hardness for a single pulse resistance spot weld with a 5 cycle hold time, calculated with **r** = 0.961.

Source: KHAN, M.I. et alii.: *Microstructure and Mechanical Properties of Resistance Spot Welded Advanced High Strength Steels*. **Materials Transactions**, 49:7, 2008, 1629-1637.

. Shinozaki

$$HV = 78 + 331\, C_{EQ_FBW}$$

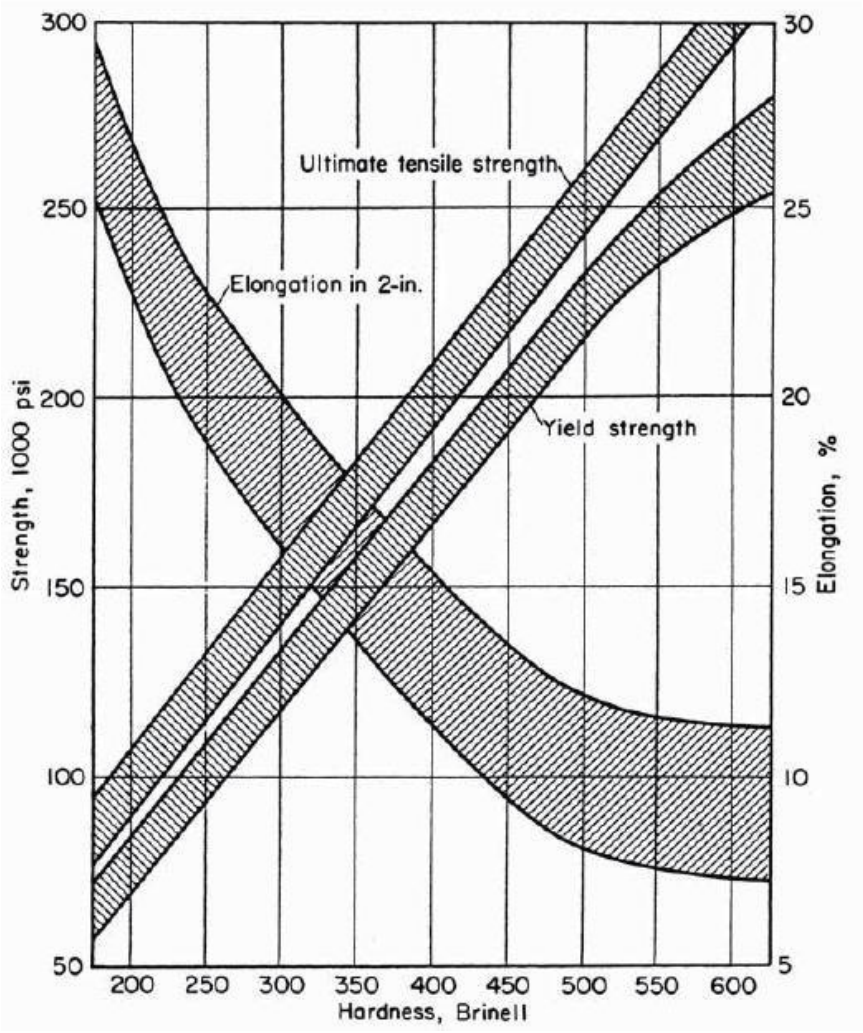
$$C_{EQ_FBW} = C + \frac{Mn}{5} + \frac{Si}{15} + \frac{Cr}{9} + 7\,Nb\,(1 - 10C) + \frac{V\,(50\,C - 1)}{3} + 1.3\,Ti\,(1 - 5\,C) + \frac{Mo\,(1 - 6\,C)}{2} + 29\,B\,(11\,C - 1)$$

Notation:

C_{EQ_FBW}: Equivalent Carbon Designed Specifically for Flash Butt Welding [%]
Alloy Content: [weight %]
HV: Hardness at the Welding Interface [Vickers]

Source: SHINOZAKI, M. et alii.: *Effects of Chemical Composition and Structure of Hot Rolled High Strength Steel Sheets on the Formability of Flash Butt Welded Joints*. **Kawasaki Steel Technical Report**, 6, Sept. 1982, 21-30.

- **Hardness-Tensile Properties Equivalence**



Source: *ASM Handbook – Mechanical Testing and Evaluation*. ASM International, vol. 8, Metals Park, 2000, 275.

. Hashemi

$$YS = 2\,HV + 105$$

$$UTS = 1.3\,HV + 344$$

Notation:
YS: Yield Strength [MPa]
UTS: Ultimate Tensile Strength [MPa]
HV: Hardness [Vickers]

Observations:
- Equations determined from an API X65 gas pipe steel with 0.071% C, 1,515% Mn, 0.209% Si, 0.044% Nb, 0.017% Ti, 0.042% V and 0.157% Cr.

Source: HASHEMI, S.H.: *Strength-Hardness Statistical Correlation in API X65 Steel*. **Materials Science and Engineering A**, 528, 2011, 1648-1655.

. Hajibagheri

$$YS = 0.5915\,HV_{10} + 406.79$$

$$UTS = 0.6788\,HV_{10} + 486.44$$

Notation:
YS: Yield Strength [MPa]
UTS: Ultimate Tensile Strength [MPa]
HV_{10}: Hardness [Vickers, 10 kg Load]

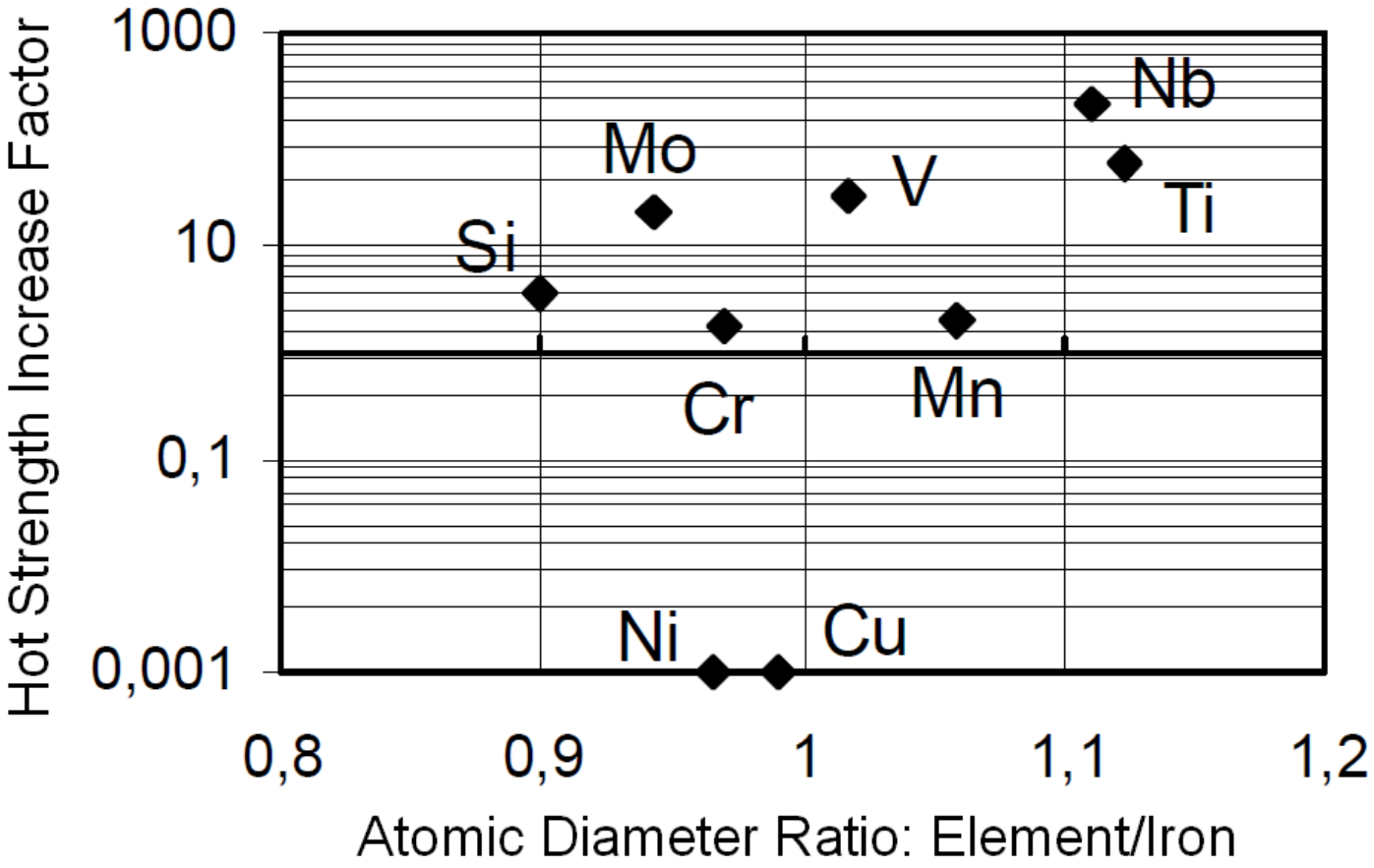
Observations:

- Equations determined from an API X70 gas pipe steel with 0.050-0.100% C, 1.45-1.60% Mn, 0.17-0.37% Si, 0.030-0.040% Nb, 0.020-0.050% Ti, 0.007-0.050% V, 0.04-0.20% Cr and 0.001-0.008% Mo.

Source: HAJIBAGHERI, H.R. et alii: *Experimental Relationship of Yield and Tensile Strengths with Hardness in High Strength API X70 Steel Pipes*. **Modares Mechanical Engineering**, 19:1, 2019, 85-93.

- **Hot Strength of Steel**

. Alloy Elements Effect



Alloy Element	$\frac{\Delta\sigma_{0.3}}{\sigma_{0.3}}$ **[%/% atomic]**	$\frac{\Delta\sigma_{0.3}}{\sigma_{0.3}}$ **[%/% weight]**
Mn	0,9838	2.13
Si	0.5054	7.52
Cr	0.9317	1.93
Mo	1.70061	12.31
Cu	0.000	0.000
Ni	0.000	0.000
Nb	1.6527	127.06
V	0.9130	31.76
Ti	0.8584	59.76

Observations:

- Plot generated from data available in the original source.

Source: TAMURA, I. et alii.: **Thermomechanical Processing of High Strength Low Alloy Steels**. Butterworths, London, 1988, 248 p.

. Misaka

$$\bar{\sigma} = \exp\left[0.126 - 1.75\,C + 0.594\,C^2 + \frac{\left(2851 + 2968\,C - 1120\,C^2\right)}{T}\right] \varepsilon^{0,21} \left(\frac{d\varepsilon}{dt}\right)^{0,13}$$

Notation:

σ: Steel Mean Flow Stress [kgf/mm²]
C: C content [weight %]
T: Absolute Temperature [K]
ε: True Strain
$\dot{\varepsilon}$: Strain Rate [s^{-1}]

Observations:
- The mean flow stress calculated by this equation is given in effective (von Mises) units, as it was determined under plane strain conditions.
- Equation valid for the following parameter range: **C** ≤ 1.20%; 750 ≤ **$\dot{\varepsilon}$** ≤ 1200°C; **ε** ≤ 0.5; 20 ≤ **$\dot{\varepsilon}$** ≤ 200 s^{-1}.

Source: MISAKA, Y. et alii. *Formulatization of Mean Resistance to Deformation of Plain C Steels at Elevated Temperature*. **Journal of the Japan Society for the Technology of Plasticity**, 8:79, 1967-1968, 414-422.

. Misaka Reloaded

$$\bar{\sigma} = f\ g \exp\left[0.126 - 1.75\,C + 0.594\,C^2 + \frac{\left(2851 + 2968\,C - 1120\,C^2\right)}{T}\right] \varepsilon^{0,21} \left(\frac{d\varepsilon}{dt}\right)^{0,13}$$

$$f = 0{,}916 + 0{,}18\,Mn + 0{,}389\,V + 0{,}191\,Mo + 0{,}004\,Ni$$

If **T**, expressed in Celsius degrees, is between $\mathbf{Ar_3}$ and $\mathbf{Ar_1}$, then **g** must be calculated according to the formula below. Otherwise **g** is equal to unity.

$$g = 0{,}7893 + 0{,}769\,C$$

$$Ar_3 = 974{,}76 - 734{,}65\,C$$

$$Ar_1 = 876{,}81 - 336{,}26\ C$$

Notation:

σ: Steel Mean Flow Stress [kgf/mm²]
f: Effect of alloy elements on Mean Flow Stress.
g: Softening Factor Due to Intercritical Deformation
C: Carbon Content [weight %]
T: Absolute Temperature [K]
ε: True Strain
t: Time [s]
Mn: Manganese Content [weight %]
V: Vanadium Content [weight %]
Mo: Molybdenum Content [weight %]
Ni: Nickel Content [weight %]
Ar_3: Temperature of Start Austenite Transformation in Proeutectoid Ferrite [°C]
Ar_1: Temperature of Finish Austenite Transformation in Proeutectoid Ferrite [°C]

Observations:

- The mean flow stress calculated by this equation is given in effective (von Mises) units, as it was determined under plane strain conditions.

Source: MISAKA, Y. et alii. *Estimation of Rolling Force in Computer Controlled Hot Rolling of Plates and Strip - Theme III: Mathematical Model for Estimating Deformation Resistance in Hot Rolling of Steels*. **Tetsu-to-Hagané**, 67:2, 1981, A53-A56.

. Senuma & Yada

$$\overline{\sigma} = a\sqrt{\rho}$$

$$\rho = \rho_n\ (1 - X_{dyn}) + \rho_s\ X_{dyn}$$

$$\rho_n = \frac{c\ (1 - e^{-b\,\varepsilon})}{b} + \rho_0\ e^{-b\,\varepsilon}$$

$$b = 9850\ \dot{\varepsilon}^{-0.315}\ e^{\left(\frac{-8000}{T}\right)}$$ (Senuma 1984)

$$b = 6227\ \dot{\varepsilon}^{-0.28}\ e^{\left(\frac{-7500}{T}\right)}$$ (Wang & Tseng)

$$c = 1.0 \times 10^{11}$$ (Senuma 1984)

$$c = 8.5 \times 10^{10} \left(1 + \frac{1}{\sqrt{D_0}}\right)$$ (Yada & Senuma)

$$\varepsilon_c = 4.76 \times 10^{-4}\ e^{\left(\frac{8000}{T}\right)}$$ (Senuma)

$$\varepsilon_c = 0.05\, e^{\left(\frac{2500}{T}\right)}$$ (Wang & Tseng 1996)

If $\varepsilon \geq \varepsilon_c$:

$$X_{dyn} = 1 - e^{\left\{-0.693\left[\frac{(\varepsilon - \varepsilon_C)}{\varepsilon_{0.5}}\right]^2\right\}}$$

$$\rho_s = 3.8242 \times 10^9 \ \dot{\varepsilon}^{0.2} \left[\frac{(1613 - T)}{290} \right]^2$$

. According to Senuma (1984):

$$\varepsilon_{0.5} = 1.144 \times 10^{-5} \ D_0^{0.28} \ \dot{\varepsilon}^{0.05} \ e^{\left(\frac{6420}{T} \right)}$$

. According to Wang & Tseng:

$$\varepsilon_{0.5} = 1.07 \times 10^{-2} \ D_0^{0.28} \ \dot{\varepsilon}^{0.03} \ e^{\left(\frac{2650}{T} \right)}$$

$$D_{dyn} = 22600 \ \dot{\varepsilon}^{-0.27} \ e^{\left(\frac{-8670}{T} \right)}$$

If $X_{dyn} > 0.95$:

. According to Yada & Senuma:

$$D_p = D_{dyn} + (D_{pd} - D_{dyn}) \left[1 - e^{-295 \dot{\varepsilon}^{0.1} e^{\left(\frac{8000}{T} \right)} t} \right]$$

. According to Wang & Tseng:

$$D_p = D_{dyn} + 1.1\,(D_{pd} - D_{dyn})\left[1 - e^{-295\,\dot{\varepsilon}^{0.1}\,e^{\left(-\frac{8000}{T}\right)}\,t}\right]$$

$$D_{pd} = 5380\,e^{\left(\frac{-6840}{T}\right)}$$

If $\varepsilon < \varepsilon_c$:

$$D_{st} = \frac{5}{(\varepsilon\,S_V)^{0.6}}$$

$$S_V = \frac{24\,(0.4914\,e^{\varepsilon} + 0.155\,e^{-\varepsilon} + 0.1433\,e^{-3\varepsilon})}{\pi\,D_0}$$

$$X_{st} = 1 - e^{\left\{-0.693\left[\frac{(t - t_s)}{t_{0.5}}\right]^2\right\}}$$

. According to Senuma (1984):

$$t_{0.5} = \frac{0.286\times10^{-7}}{\sqrt{S_v}}\,\dot{\varepsilon}^{-0.2}\,\varepsilon^{-2}\,e^{\left(\frac{18000}{T}\right)}$$

. According to Wang & Tseng:

$$t_{0.5} = \frac{2.2 \times 10^{-12}}{\sqrt{S_v}} \dot{\varepsilon}^{-0.2} \varepsilon^{-2} e^{\left(\frac{30000}{T}\right)}$$

$$t_{0.95} = 4.322\, t_{0.5}$$

If $X_{st} < 0.95$ (i.e., $t_{ip} < t_{0.95}$):

$$D_u = D_{st} (0.2 + 0.8\, X_{st})$$

If $X_{st} \geq 0.95$ and $t_{ip} \geq t_{0.95}$:

. According to Senuma (1984):

$$D_g = \sqrt{D_{st}{}^2 + 1.44 \times 10^{12}\, t_g\, e^{\left(\frac{63800}{T}\right)}}$$

. According to Wang & Tseng:

$$D_g = \sqrt{D_{st}{}^2 + 1.44 \times 10^{12}\, t_g\, e^{\left(\frac{32100}{T}\right)}}$$

$$t_g = t_{ip} - t_{0.95}$$

In case of multipass hot rolling:

$$\rho_R = \rho_t\,(1 - X_{dyn})\,(1 - X_{st}) + \rho_s\; X_{dyn}\,\frac{(D_{pd} - D_p)}{(D_{pd} - D_{dyn})}$$

$$\rho_t = \rho_n\; e^{\left[-90\, e^{\left(\frac{-8000}{T}\right)} t_a{}^{0.7}\right]}$$

$$\varepsilon_r = \frac{\ln\left[\dfrac{(\rho_0\, b - c)}{(\rho_r\, b - c)}\right]}{b}$$

Notation:

σ: Steel Mean Flow Stress [kgf/mm²]
ρ: Dislocation Density [cm^{-2}]
ρ_0: Initial Dislocation Density [cm^{-2}]
ρ_n: Dislocation Density in the Dynamically Recovered Region [cm^{-2}]
ρ_s: Dislocation Density in the Dynamically Recristallized Region [cm^{-2}]
ρ_r: Dislocation Density After Deformation/Static Recovery [cm^{-2}]
ρ_t: Remaining Dislocation Density in the Dynamically Recovered Region [cm^{-2}]
X_{dyn}: Fraction of Dynamic Recrystallization
X_{st}: Fraction of Static Recrystallization
T: Absolute Temperature [K]
ε: True Strain
ε_c: Critical Strain for the Onset of Dynamic Recrystallization
$\varepsilon_{0.5}$: Strain Required for 50% Dynamic Recrystallization
ε_r: Residual Strain After One Pass of Hot Rolling

$\dot{\varepsilon}$: Strain Rate [s^{-1}]
$\mathbf{D_0}$: Grain Size Before Deformation [µm]
$\mathbf{D_{dyn}}$: Dynamically Recrystallized Grain Size [µm]
$\mathbf{D_p}$: Transition Grain Size from $\mathbf{D_{dyn}}$ to $\mathbf{D_{pd}}$ at a time **t** after deformation [µm]
$\mathbf{D_{pd}}$: Grain Size resulted from driving force due to the decrease of dislocation density [µm]
$\mathbf{D_{st}}$: Statically Recrystallized Grain Size [µm]
$\mathbf{D_u}$: Mixed Grain Size Due to Incomplete Static Recrystallization [µm]
$\mathbf{D_g}$: Grain Size After Complete Static Recrystallization Plus Growth [µm]
$\mathbf{S_v}$: Nucleation Site Area [μm^{-1}]
t: Time [s]
$\mathbf{t_s}$: Incubation Time for Static Recrystallization [s]
$\mathbf{t_{0.5}}$: Time Required for 50% Static Recrystallization [s]
$\mathbf{t_{0.95}}$: Time Required for 95% Static Recrystallization [s]
$\mathbf{t_{ip}}$: Time Interval Between Successive Rolling Passes [s]
$\mathbf{t_g}$: Time Available for Grain Growth [s]
$\mathbf{t_a}$: Time After Deformation [s]

Observations:

- This model is very interesting as it links hot strength with microstructural evolution.
- The mean flow stress calculated by this equation is given in effective (von Mises) units, as it was determined under plane strain conditions.
- The value of constant **a** depends on steel composition. For instance:
  . 0.00175 MN/m^2 (Senuma 1984: 0.08-0.81% C, 0.62-1.14% Mn, 0.20-0.24% Si)
  . 0.00165 MN/m^2 (Yada & Senuma: 0.05-0.40% C, 0.00-1.00% Mn, 0.00-0.50% Si)
  . 0.00180 MN/m^2 (Wang & Tseng: 0.05-0.81% C, 0.20-1.50% Mn, 0.01-0.50% Si)
- Suggested value for $\boldsymbol{\rho_0}$: 1×10^{-8} cm^{-2} (Wang & Tseng)
- $\mathbf{t_s}$ can be assumed as being zero as it is negligibly short for the deformation conditions of hot flat rolling (Wang & Tseng).
- If $\boldsymbol{\varepsilon_r}$ from the former pass is greater than 0, then it must be added to value of $\boldsymbol{\varepsilon}$ of the next pass.

Sources:

- SENUMA, T. & YADA, H. *Microstructure Evolution of Plain Carbon Steels*. **7th Riso International Symposium on Metallurgy and Materials Science**, Riso National Laboratory, Roskilde, 1986, p. 547-552.

- WANG, S.R. & TSENG, A.A. *Macro- and Micro-Modeling of Hot Rolling of Steel Coupled by a Micro Constitutive Relationship*. **Iron and Steelmaker**, September 1996, 49-61.

- SENUMA, T. et alii. *Structure of Austenite of Carbon Steels in High Speed Hot Working Process*. **Tetsu-to-Hagané**, 70:15, November 1984, 2112-2119.

- YADA, H. & SENUMA, T. *Resistance to Hot Deformation of Steels*. **Journal of the Japan Society for Technology of Plasticity**, 27:300, 1986, 34-9.

- YANAGIMOTO, J. & LIU, J. *Incremental Formulation for the Prediction of Microstructural Change in Multi-Pass Hot Forming.* **ISIJ International**, 39:2, February 1999, 171-175.

. Shida

Calculation algorithm expressed in Visual Basic:

```
Function Shida(C, T, Def, VelDef)

Dim nShida, Td, g, Tx, mShida, SigF As Single

   nShida = 0.41 – 0.07 * C
   Td = 0.95 * (C + 0.41) / (C + 0.32)
   T = (T + 273) / 1000
   If T >= Td Then
      g = 1
      Tx = T
      mShida = (-0.019 * C + 0.126) * T + (0.075 * C – 0.05)
           Else
```

```
    g = 30 * (C + 0.9) * (T – 0.95 * (C + 0.49) / (C + 0.42)) ^ 2 + (C + 0.06) / (C + 0.09)
    Tx = Td
    mShida = (0.081 * C – 0.154) * T + (-0.019 * C + 0.207) + 0.027 / (C + 0.32)
  End If
  SigF = 0.28 * g * Exp(5 / Tx – 0.01 / (C + 0.05))
  Shida = 2 / Sqr(3) * SigF * (1.3 * (Def / 0.2) ^ nShida – 0.3 * (Def / 0.2)) * _
      (VelDef / 10) ^ mShida

End Function
```

Notation:

Shida: Steel Mean Flow Stress [kgf/mm²]
C: C content [weight %]
T: Temperature [°C]
Def: True Strain
VelDef: Strain Rate [s^{-1}]

Observation:

- The mean flow stress calculated is this algorithm is already expressed in effective (von Mises) units, that is, corrected for plane strain conditions, as it is multiplied by $2/\sqrt{3}$.
- Equation valid for the following parameter range: $\mathbf{C} \leq 1.20\%$; $700 \leq \dot{\boldsymbol{\varepsilon}} \leq 1200°C$; $\boldsymbol{\varepsilon} \leq 0.7$; $0.1 \leq \dot{\boldsymbol{\varepsilon}} \leq 100\ s^{-1}$.
- The effect of some alloy elements over hot strength can be considered by Shida equation. In this case carbon content must be replaced by an equivalent carbon ($\mathbf{C_{eq}}$) content, which formula is described below:

$$C_{eq} = C + \frac{Mn}{6} + \frac{Cr + V + Nb}{12}$$

where **Mn** is the manganese content, **Cr** is the chromium content, **V** is the vanadium content and **Nb** is the niobium content, all expressed as weight percent.

Sources:

- SHIDA, S. *Empirical Formula of Flow Stress of C Steels - Resistance to Deformation of C Steels at Elevated Temperature.* **Journal of the Japan Society for Technology of Plasticity**, 10:103, 1969, 610-7.

- LENARD, J.G. et alii. : **Mathematical and Physical Simulation of the Properties of Hot Rolled Products**. Elsevier, Amsterdam, 1999, 248 p.

- **Jominy Curves**

. Just

- d < 6,4 mm

$$J_d = 60\sqrt{C} + 20$$

- 6,4 ≤ d < 39,7 mm

$$J_d = 98\sqrt{C} + 0.00992\ d^2\sqrt{C} + 20\ Cr + 6.4\ Ni + 19\ Mn + 34\ Mo + 28\ V - 19.05\sqrt{d} + 1.80\ d + 7$$

- C < 0.28% and 6,4 mm ≤ d < 39,7 mm

$$J_d = 87\ C + 14\ Cr + 5.3\ Ni + 16\ Mn + 29\ Mo - 16.8\sqrt{d} + 1.39\ d + 22$$

- C > 0.29% and 6,4 mm ≤ d < 39,7 mm

$$J_d = 78\ C + 22\ Cr + 6.9\ Ni + 21\ Mn + 33\ Mo - 16.1\sqrt{d} + 1.17\ d + 18$$

Observation:

- Equations valid for the following chemical composition range: 0.10% ≤ C ≤ 0.60%, 0.45% ≤ Mn ≤ 1.75%, 0.15% ≤ Si ≤ 1.95%, Ni ≤ 5.0%, Cr ≤ 1.55%, Mo ≤ 0.52% and V ≤ 0.2%.

- Equation Considering the Effect of Austenite Grain Size

$$J_d = 88\sqrt{C} - 0.00553\sqrt{C} + 19\,Cr + 6.3\,Ni + 16\,Mn + 35\,Mo + 5\,Si - 0.82\,G_\gamma - 15.9\sqrt{d} + 1.33\,d - 2$$

Observation:

- Equation valid for the following conditions: 0.08% ≤ C ≤ 0.56%, 0.20% ≤ Mn ≤ 1.88, Si ≤ 3.80, Ni ≤ 8.94%, Cr ≤ 1.97, Mo ≤ 0.53 and 1.5 ≤ Gγ ≤ 11.

Notation:

J_d: Hardness [Rockwell C]
Alloy Content: [weight %]
d: Distance from the Cooled End [mm]
G_γ: Austenite Grain Size Index [mm]

Source: JUST, E. *New Formulas for Calculating Hardenability.* **Metal Progress**, 96, November 1969, 87-88.

- **Lattice Parameters of Phases**

. Ferrite

$$a_{\alpha} = 2.8863\ [1 + 17.5 \times 10^{-6}\ (T - 800)]$$

Notation:

a_{α}: Ferrite Lattice Parameter [Å]
T: Temperature [K]

Observations:

- 800 K < T < 1200 K

. Austenite

$$a_{\gamma_0} = 3.573 + 0.033\ C + 0.00095\ Mn - 0.0002\ Ni + 0.0006\ Cr + 0.0031\ Mo + 0.0018\ V$$

Notation:

a_{γ}: Austenite Lattice Parameter [Å]
T: Temperature [K]
ξ: C [Atomic Fraction]

Observations:

- 1000 K < T < 1250 K
- 0.0005 < ξ < 0.0365

$$a_{\gamma} = (3.6306 + 0.78\ \xi)\ [1 + (24.9 - 50\ \xi) \times 10^{-6}\ (T - 1000)]$$

Notation:

a_γ: Austenite Lattice Parameter [Å]
Alloy Content: [Weight Percent]

Observations:

- 1000 K < T < 1250 K

. Cementite

$$a_\theta = 4.5234\ [1 + (5.311 \times 10^{-6} - 1.942 \times 10^{-9}\ T + 9.655 \times 10^{-12}\ T^2)\ (T - 293)]$$

$$b_\theta = 5.0883\ [1 + (5.311 \times 10^{-6} - 1.942 \times 10^{-9}\ T + 9.655 \times 10^{-12}\ T^2)\ (T - 293)]$$

$$c_\theta = 6.7426\ [1 + (5.311 \times 10^{-6} - 1.942 \times 10^{-9}\ T + 9.655 \times 10^{-12}\ T^2)\ (T - 293)]$$

Notation:

a_θ, b_θ, c_θ: Cementite Lattice Parameter [Å]
T: Temperature [K]

Observations:

- 300 K < T < 1000 K

Source: CABALLERO, F.G. et alii. *Modelling of Kinetics and Dilatometric Behaviour of Austenite Formation in a Low-carbon Steel with a Ferrite Plus Pearlite Inicial Microstructure.* **Journal of Materials Science**, 37, 2002, 3533-3540.

- **Liquid Steel Solubility Products**

. General

$$\log \frac{(a_A)^m \, (a_B)^n}{a_{A_m B_n}} = -\frac{A}{T} + B$$

Notation:

A_mB_n: Precipitate Considered for Calculation
a_x: Alloy Content [weight %]
T: Temperature [K]
A, B: Constants of the Solubility Product, given in the table below:

Precipitate	A	B
MnS	8236	5.03
TiN	16586	5.90
TiS	8000	4.00
ZrN	17000	6.38

Observations:

- a_{AmBn} is equal to one if the precipitate is pure.
- $a_{AmBn} \leq 1$ if there is co-precipitation with another element.

Source: Values compiled by Rajindra Clement Ratnapuli from assorted references.

- **Liquidus Temperature of Steels**

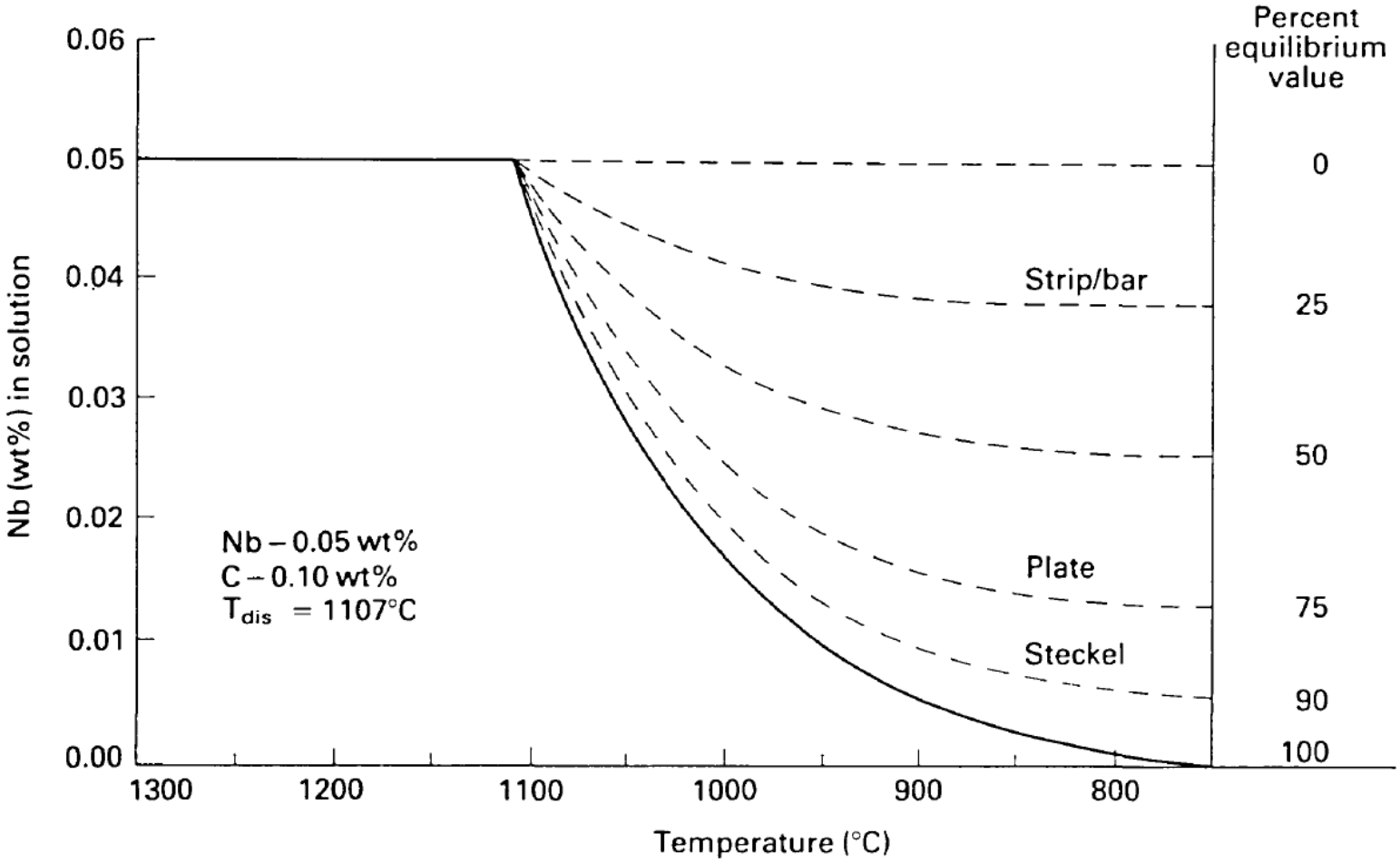
$$T_{Liq} = 1536 - (78\,C + 7.6\,Si + 4.9\,Mn + 34\,P + 30\,S + 5\,Cu + 3.1\,Ni + 1.3\,Cr + 3.6\,Al + 2\,Mo + 2V + 18\,Ti)$$

Notation:

$T_{Líq}$: Steel Melting Temperature [°C]
Alloy Content: [weight %]

Source: GUTHMANN, K. *Günstige Giesstemperatur im Vergleich zum Erstarrungspunkt von Eisen- und Stahlschmelzen*. **Stahl und Eisen**, 71: 8, 1951, 399-402.

- **Niobium Carbide Precipitation During Rolling**



Source: DeARDO, A.J. *Accelerated Cooling: A Physical Metallurgy Perspective*. **Canadian Metallurgical Quarterly**, 27: 2, 1988, 141-154.

- **Poisson Ratio**

. Definition

υ: Poisson Ratio
. Elastic Range: 0.3
. Plastic Range: 0.5

Source: WILSON, A.D. *Guidelines for Fabricating and Processing Plate Steel*. Bethlehem-Lukens Plate Report, Burns Harbor, 2000, 97 p.

. Fletcher

Temperature [°C]	υ
600	0.327
700	0.335
800	0.344
900	0.352
1000	0.360

Source: PICQUÉ, B. *Experimental Study and Numerical Simulation of Iron Oxide Scales Behavior in Hot Rolling*. **Doctor Thesis**, École de Mines de Paris, 2004, p. 243.

- **Precipitate Isothermal Solubilization Kinetics**

$$t = \frac{r_0^2}{2\,c\,D}$$

Notation:

A_mB_n: Spheric precipitate considered for calculation
t: Time for solubilization of the precipitate [s]
r_0: Radius of the precipitate [m], [cm] or [mm]

$$c = \frac{C_i - C_m}{C_p - C_i} \cong \frac{C_i}{C_p}$$

C_m: Solute concentration in the bulk metal [%]
C_i: Solute concentration in the precipitate/matrix interface [%]

$$C_i = \frac{10^{\left(\frac{-A}{T} + B\right)}}{a_B}$$

T: Temperature [K]
A, B: Constants of the Solubility Product, given in the table at the topic *Austenite Solubilization Products*.
a_B: Alloy content [weight percent]
C_p: Solute content in the precipitate [%]

$$C_p = \frac{m\,M_A}{m\,M_A + n\,M_B}$$

M_x : Atomic mass of the element [g]

C_m: Solute content in a position far away from the precipitate [%]
D: Solute Diffusion Coefficient [m²/s, cm²/s or mm²/s], calculated according to the general equation below:

$$D = D_0 \exp\left(\frac{-Q}{RT}\right)$$

D_0: Constant
Q: Activation Energy for Diffusion [J] or [cal]
R: Universal Gas Constant, 1.981 cal/mol.K
R': Universal Gas Constant, 8.314 J/mol.K

Element	Phase	Equation	Source
Al	Ferrite	D [m^2/s] = 0.30 * 10^{-2} * Exp(-234500/R' T)	Gladman
	Austenite	D [m^2/s] = 0.49 * 10^{-4} * Exp(-284100/R' T)	
	Austenite	D [m^2/s] = 2.10 * 10^{-3} * Exp(-286000/R' T)	Borggren
B	Austenite	D [m^2/s] = 2 * 10^{-4} * Exp(-87864/R'T)	
C	Ferrite	D [cm^2/s] = 0.02 * Exp(-20100/RT)	
	Ferrite	D [m^2/s] = 0.62 * 10^{-6} * Exp(-80400/R'T)	Gladman
	Austenite	D [m^2/s] = 0.10 *10^{-4} * Exp(-135700/R'T)	
Cr	Ferrite	D [cm^2/s] = 8.52 * Exp(-59900/RT)	

	Austenite	$D\ [cm^2/s] = 10.80 * Exp(-69700/RT)$	
Fe	Ferrite	$D\ [m^2/s] = 1.67 * 10^{-4} * Exp(-256700/R'T)$	Gladman
	Austenite	$D\ [m^2/s] = 0.49 * 10^{-4} * Exp(-284100/R'T)$	Gladman
	Austenite	$D\ [m^2/s] = 7.00 * 10^{-5} * Exp(-28600/R'T)$	Borggren
Mn	Austenite	$D\ [mm^2/s] = 140 * Exp(-286000/R'T)$	
	Austenite	$D\ [cm^2/s] = 0.65 * Exp(-276000/R'T)$	
	Austenite	$D\ [m^2/s] = 1.78 * 10^{-5} * Exp(-264000/R'T)$	Borggren
N	Ferrite	$D\ [cm^2/s] = 6.6 * 10^{-3} * Exp(-18600/RT)$	
	Ferrite	$D\ [m^2/s] = 0.50 * 10^{-6} * Exp(-77000/R'T)$	Gladman
	Austenite	$D\ [m^2/s] = 0.91 * 10^{-4} * Exp(-168600/R'T)$	Gladman
Nb	Austenite	$D\ [mm^2/s] = 5.90 * 10^{-4} * Exp(-343000/R'T)$	Andersen
	Austenite	$D\ [m^2/s] = 5.30 * 10^{-2} * Exp(-344600/R'T)$	Gladman
	Austenite	$D\ [m^2/s] = 5.60 * 10^{-4} * Exp(-286000/R'T)$	Borggren
P	Austenite	$D\ [mm^2/s] = 51 * Exp(-230120/R'T)$	
	Austenite	$D\ [cm^2/s] = 2.90 * Exp(-55000/RT)$	

Si	Austenite	D [m^2/s] = 7.00 * 10^{-4} * Exp(-286000/R' T)	Borggren
Ti	Austenite	D [m^2/s] =1.50 * 10^{-5} * Exp(-251000/R'T)	Borggren
V	Ferrite	D [cm^2/s] = 3.92 * Exp(-57600/RT)	
	Ferrite	D [m^2/s] = 0.61 * 10^{-4} * Exp(-267100/R' T)	Gladman
	Austenite	D [cm^2/s] = 0.25 * Exp(-63100/RT)	
	Austenite	D [m^2/s] = 0.25 * 10^{-4} * Exp(-264200/R' T)	Gladman

Sources:

- ANDERSEN, I. & GRONG, O. *Analytical Modelling of Grain Growth in Metals and Alloys in the Presence of Growing and Dissolving Precipitates – I. Normal Grain Growth*. **Acta Metallurgica and Materialia**, 43:7, 1995, 2673-2688.

- GLADMAN, T. *The Physical Metallurgy of Microalloyed Steels*. The Institute of Materials, London, 1997, p. 206

- BORGGREN, U. e al. *A Model for Particle Dissolution and Precipitation in HSLA Steels*. **Advanced Materials Research**, 15-17, 2007, 714-719.

- Information compiled by Rajindra Clement Ratnapuli from assorted references.

- **Relationships Between Chemical Composition x Process x Microstructure x Properties**

. Acicular Ferrite/Low Carbon Bainite Steels

$$YS = 88 + 37\ Mn + 83\ Si + 2900\ N_{sol} + \frac{15.1}{\sqrt{d_L}} + \sigma_{disc} + \sigma_{ppt}$$

$$TS = 246 + 1900\ C + 230\ (Mn + Cr) + 185\ Mo + 90\ W + 125\ Ni + 65\ Cu + 385\ (V + Ti)$$

$$ITT = -19 + 44\ Si + 700\ \sqrt{N_{sol}} + 0.26\ (\sigma_{disc} + \sigma_{ppt}) - \frac{11.5}{\sqrt{d}}$$

Notation:

YS: Yield Strength at 0.2% Real Strain [MPa]
TS: Tensile Strength [MPa]
ITT: Impact Transition Temperature for 50% Tough Fracture [°C]
Alloy Content: [weight %]
$\mathbf{N_{sol}}$: Solubilized (Free) Nitrogen [%]
$\mathbf{d_L}$: BainiteFerrite Lath Size [mm]
$\boldsymbol{\sigma_{disc}}$: Strength Due to Dislocations [MPa]
$\boldsymbol{\sigma_{ppt}}$: Precipitation Strengthening According to the Ashby-Orowan Model [MPa]
$\mathbf{N_{sol}}$: Solubilized (Free) Nitrogen [%]
d: Mean Spacing between High Angle Boundaries (“Packet” or Prior Austenite Grain Boundaries)

$$\Delta\sigma_{disc} = \alpha\ \mu\ b\ \sqrt{\rho} = 1.2 \times 10^{-3}\ \sqrt{\rho}\ (PICKERING)\ or\ 8 \times 10^{-4}\ \sqrt{\rho}\ (KEH)$$

$$\Delta\sigma_{ppt} = \frac{5.9\ \sqrt{f}}{\bar{x}}\ \ln\left(\frac{\bar{x}}{2.5 \times 10^{-4}}\right)$$

Notation:

α: Empirical Constant
μ: Shear Modulus [MPa]
b: Burger's Vector [cm]
ρ: Dislocation Density [lines/cm²]
f: Volume Fraction of the Precipitate
x: Mean Planar Intercept Diameter of the Precipitate [μm]

Sources:

- PICKERING, F.B. *Some Aspects of the Relationships between the Mechanical Properties of Steels and their Microstructures.* **TISCO**. Silver Jubilee Volume, Jan-Oct 1980, 105-132.

- KEH, A.S., *Work Hardening and Deformation Sub-Structure in Iron Single Crystals in Tension at 298K*, **Philosophical Magazine**, 12:115, 1965, 9-30.

. C-Mn Mild Steels (Pickering)

$$YS = 53.9 + 32.3\ Mn + 83.2\ Si + 354.2\ \sqrt{N_{sol}} + \frac{17.4}{\sqrt{d}}$$

$$TS = 294.1 + 27.7\ Mn + 83.2\ Si + 2.85\ Pearl + \frac{7.7}{\sqrt{d}}$$

$$\frac{d\sigma}{d\varepsilon} = 370 + 120\,C + 23.1\,Mn + 116\ Si + 554\ P + 143\ Sn + 1509\ N_{sol} + \frac{15.4}{\sqrt{d}}$$

$$\varepsilon_{unif} = 0.28 - 0.20\ C - 0.25\ Mn - 0.044\ Si - 0.039\ Sn - 1.2\ N_{sol}$$

$$\varepsilon_{tot} = 1.40 - 2.90\,C + 0.20\,Mn + 0.16\,Si - 2.2\,S - 3.9\,P + 0.25\,Sn + \frac{0.017}{\sqrt{d}}$$

$$50\%\ ITT = -19 + 44\,Si + 700\,\sqrt{N_{sol}} + 2.2\,Pearl - \frac{11.5}{\sqrt{d}}$$

$$\Delta Y = 12.32 - 19250\,N_{sol} + 162\,Mn + 462\,O$$

Notation:

YS: Yield Strength at 0.2% Real Strain [MPa]
TS: Tensile Strength [MPa]
dσ/dε: Strain Hardening Coefficient at 0.2% Real Strain [1/MPa]
ε_{unif}: Uniform Elongation, Expressed as Real (Logarithmic) Strain
ε_{tot}: Total Elongation, Expressed as Real (Logarithmic) Strain
Pearl: Pearlite Fraction in Microstructure [%]
50% ITT: Impact Transition Temperature for 50% Tough Fracture [°C]
ΔY: Strain Ageing After 10 Days at Room Temperature [MPa]
Alloy Content: [weight %]
d: Grain Size [mm]

Source: PICKERING, F.B.: **Physical Metallurgy and the Design of Steels**. Allied Science Publishers, London, 1978, 275 p.

. C-Mn Mild Steels (Choquet)

$$YS = 63 + 23\,Mn + 53\,Si + 700\,P + \frac{\left(\frac{15.4 - 30\,C + 6.094}{0.8 + Mn}\right) F}{\sqrt{d}} + (360 + 2600\,C^2)\,(1 - F)$$

$$TS = 237 + 29\ Mn + 79\ Si + 700\ P + \frac{7.24\ F}{\sqrt{d}} + 500\ (1 - F)$$

Notation:

YS: Yield Strength at 0.2% Real Strain [MPa]
TS: Tensile Strength [MPa]
Alloy Content: [weight %]
F: Ferrite Fraction
d: Ferritic Grain Size [mm]

Source: CHOQUET, P. et alii.: *Modelling of Forces, Structure and Final Properties during the Hot Rolling Process on the Hot Strip Mill*. In: **Mathematical Modelling of Hot Rolling of Steels**. Proceedings. The Metallurgical Society of CIM, Montreal, 1990, 34-43.

. C-Mn Steels Processed at a Hot Strip Mill

$$d = 11.5 - 2.2\ (6\ C + Mn + 30\ P + 35\ S + 23\ Al + 0.01\ (723 - T_{coil}) + 0.01\ e_{tot} - 0.002\ T_{fin} - 100\ N_{sol})$$

$$Pearl = \alpha\ \frac{C_{eq} - 0.06}{0.78}\ 100$$

$$S_0 = \frac{0.1}{723 - T_{coil}}$$

$$YS = 99.08\ (38.2 + \frac{0.016\ Pearl}{\sqrt{S_0}} + 5.5\ Mn + 43\ Si + 114\ P - 45\ S + 31\sqrt{N_{sol}} + \frac{12.6}{\sqrt{d}} - 0.02\ T_{fin})$$

$$TS = 130.47\,(19.8 + \frac{0.004\,Pearl}{\sqrt{S_0}} + 8.03\,Mn + 41.4\,Si + 57.7\,P - 69\,S + 262\,\sqrt{N_{sol}} + \frac{11.5}{\sqrt{d}})$$

$$\varepsilon = 100\,(0.000096\,Pearl\,S_0 - 0.05\,Mn - 4.23\,P - 4.36\,S + 2.37\,Sn - 1.16\,\sqrt{N_{sol}} + \frac{0.12}{\sqrt{d}} + 0.0006\,T_{fin})$$

Notation:

YS: Yield Strength at 0.2% Real Strain [MPa]
TS: Tensile Strength [MPa]
ε: Total Elongation [%]
d: Ferrite Grain Size [μm]
Alloy Content: [weight %]
T_{coil}: Coiling Temperature [°C]
e_{tot}: Total Hot Rolling Conventional Strain [%]
T_{fin}: Finishing Temperature [°C]
N_{sol}: Solubilized (Free) Nitrogen [%]
Pearl: Pearlite Fraction Present in Microstructure [%]
S_0: Pearlite Lamelar Spacing [mm]

$$C_{eq} = C + \frac{Mn}{6} + \frac{Si}{24} - S$$

$$\alpha = \frac{T_{fin} - T_{coil}}{T_{fin}}$$

Observations:

- These equations are valid under the following conditions: **Slab Reheating Temperature**: 1250°C; **T_{fin}**: 850~880°C; **T_{coil}**: 615~650°C; **Final Thickness**: 1.8~4.0 mm; **C**: 0.08~0.18%; **Mn**: 0.40~1.00%; **P** < 0.020%; **S** < 0.020%; **Si** < 0.030%; **Al**: 0.020~0.050%; **N**: 0.0030~0.0090%.

Source: ARTIGAS, A. et alii.: *Prediction de Propiedades Mecánicas y Microestructurales em Aceros Laminados en Caliente*. **Revista Metalurgica CENIM**, 38, 2002, 339-347.

. C-Mn Mild Steel: Hot/Cold Rolled and Annealed

$$d_{CR} = 0.013 + 0.28\, d_{HR} \quad (after\ 60\%\ cold\ rolling\ and\ annealing)$$

$$d_{CR} = 0.011 + 0.29\, d_{HR} \quad (after\ 70\%\ cold\ rolling\ and\ annealing)$$

$$YS = 3.37 + \frac{2.72}{\sqrt{d_{CR}}} \qquad YS = 28.16 - 154\, d_{HR}$$

$$\varepsilon_{yield} = 6.27 - 78.5\, d_{CR}$$

$$n = 0.33 - \frac{0.01}{\sqrt{d_{CR}}}$$

Notation:

d_{CR}: Grain Size of Cold Rolled Strip [mm]
d_{HR}: Grain Size of Hot Rolled Strip [mm]
YS: Yield Strength at 0.2% Real Strain [MPa]
ε_{yield}: Yield Elongation [%]
n: Strain Hardening Coefficient Measured during Tension Test

Observations:

- These equations are valid under the following conditions: **C**: 0.005~0,10%; **Mn**: 0.40%; **P** < 0.016%; **S** < 0.026%; **Si** < 0.010%; **Al**: < 0.040%; **N**: 0.0020~0.0040%.

- Cold rolled steel was box annealed at 700°C; the time of treatment, including heating of the samples, was equal to 32 hours, being followed by furnace cooling.

Source: LANGENSCHEID, G. et alii: *Untersuchungen über den Einfluß der Korngröße des Warmbandes auf die Kaltbandeigenschaften*. **Hoesch Berichte aus Forschung und Entwicklung unserer Werke**, 2, 1971, 64-70.

. C-Mn Mild Steel, Full Annealed

$$n = \frac{5}{10 + \frac{1}{\sqrt{d}}}$$

Notation:

n: Strain Hardening Coefficient Measured during Tension Test
d: Grain Size [mm]

Source: MORRISON, W.: *The Effect of Grain Size on the Stress-Strain-Relationship in Low-Carbon Steel*. **Transactions of the ASM**, 59, 1966, 824-845.

. C-Mn Steels with Ferrite-Pearlite Structure (Grozier & Bucher)

$$YS = 95.84 + 40.68\ Mn + 70.40\ Si + 1.517\ Pearl + \frac{3.282}{\sqrt{d}}$$

$$TS = 223.11 + 56.74\ Mn + 101.97\ Si + 4.323\ Pearl + \frac{2.344}{\sqrt{d}}$$

Notation:

YS: Yield Strength [MPa]
TS: Tensile Strength [MPa]
Pearl: Pearlite Fraction in Microstructure [%]
Alloy Content: [weight %]
d: Grain Size [mm]

Observations:

- These equations were fitted using at least 50 points of data.
- Useful range: **Mn**: 0.00 ~ 1.60%; **Si** 0.00 ~ 0.80%; **Pearl**: 0 ~ 80%; **d**: 0.000252 ~ 0.002770 cm.
- 95% confidence limits: yield strength, ±26MPa; tensile strength, ±52 MPa.
- Eventually pearlite fraction can be calculated with the equation below:

$$Pearl = 10.7 + 110.9\,C + 11.3\,Mn + 48.4\,Si$$

which was fitted used 32 points of date of ferritic-pearlitic hot rolled, air cooled and normalized steel , cooled in air with a mean cooling rate of 1°C/s at 760°C. Its useful range is 0.00~0.30% **C**; 0.00~1.80% **Mn**, 0.00~0.25% **Si** and 0~40% **Pearl**. Its 95% confidence limit is ±7%; correlation coefficient **r** is equal to 0.89.

Source: GROZIER, J.D. & BUCHER, J.H.: *Influence du Niobium et de l'Azote sur la Résistance des Aciers a Structure Ferrite-Perlite*. **Revue de Métallurgie**, 63:11, Novembre 1966, 939-941.

. C-Mn Steels with Ferrite-Pearlite Structure (Pickering)

$$YS = 246 + 4.15\,Pearl + 44.6\,Mn + 138\,Si + 923\,P + 169\,Sn + 3754\,N_{sol} + \frac{14.9}{\sqrt{d}}$$

$$TS = 492 - 3.38\,Pearl + 246\,Mn + 277\,Si - 2616\,S + 723\,P + 246\,Cr + 6616\,N_{sol} + \frac{44.6}{\sqrt{d}}$$

$$\frac{d\sigma}{d\varepsilon} = 385 + 1.39\ Pearl + 111\ Si + 462\ P + 152\ Sn + 1369\ N_{sol} + \frac{15.4}{\sqrt{d}}$$

$$\varepsilon_{unif} = 0.27 - 0.016\ Pearl - 0.015\ Mn - 0.040\ Si - 0.043\ Sn - 1.0\ N_{sol}$$

$$\varepsilon_{tot} = 1{,}30 - 0.020\ Pearl + 0.30\ Mn + 0.20\ Si - 3.4\ S - 4.4\ P + 0.29\ Sn + \frac{0.015}{\sqrt{d}}$$

$$T_{trans} = 43 + 1.5\ Pearl - 37\ Mn - \frac{6.2}{\sqrt{d}}$$

Notation:

YS: Yield Strength at 0.2% Real Strain [MPa]
TS: Tensile Strength [MPa]
dσ/dε: Strain Hardening Coefficient at 0.2% Real Strain [1/MPa]
ε_{unif}: Uniform Elongation, Expressed as Real (Logarithmic) Strain
ε_{tot}: Total Elongation, Expressed as Real (Logarithmic) Strain
Pearl: Pearlite Fraction in Microstructure [%]
T_{trans}: Fracture Appearance Transition Temperature [°C]
Alloy Content: [weight %]
d: Grain Size [mm]

Source: PICKERING, F.B.: *The Effect of Composition and Microstructure on Ductility and Toughness*; in: **Towards Improved Ductility and Toughness**, Climax Molybdenum Company, Tokyo, 1971, p. 9-32

. Dual Phase Steels

$$YS = 203 + 855\sqrt{\frac{1}{L_{\alpha\alpha}}}$$

$$TS = 266 + 548\sqrt{\frac{1}{L_{\alpha\alpha}}} + 1741\sqrt{\frac{f_\beta}{d_\beta}}$$

$$\frac{d\sigma}{d\varepsilon} = 266 + 548\sqrt{\frac{1}{L_{\alpha\alpha}}} + 1741\sqrt{\frac{f_\beta}{d_\beta}}$$

$$\varepsilon_{unif} = 32 - 64\sqrt{\frac{1}{L_{\alpha\alpha}}}$$

Notation:

LE: Yield Strength [MPa]
LR: Tensile Strength [MPa]
dσ/dε: Strain Hardening Coefficient at Uniform Elongation [1/MPa]
$\mathbf{a_{unif}}$: Uniform Elongation [%]
$\mathbf{L_{\alpha\alpha}}$: Mean Ferritic Free Path [μm]
$\mathbf{d_\beta}$: Mean Diameter of Martensite Islands [μm]

Sources:

- GORNI, A.A. & BRANCHINI, O.L.G. *Análise da Evolução do Encruamento de um Aço Bifásico*. In: **4° Simpósio de Conformação Mecânica**, EPUSP/UNICAMP/ABAL, São Paulo, Nov. 1990, 23-42.

- GORNI, A.A. & BRANCHINI, O.L.G. *Relações Microestrutura-Propriedades Mecânicas em um Aço Bifásico Laminado a Quente*. In: **1º Seminário sobre Chapas Metálicas para a Indústria Automobilística**, ABM/AEA, São Paulo, Set. 1992, 127-145.

. Medium C Steels

$$YS = \sqrt[3]{f_\alpha}\left(35 + 58\,Mn + \frac{17.4}{\sqrt{d}}\right) + \left(1 - \sqrt[3]{f_\alpha}\right)\left(178 + \frac{3.8}{\sqrt{S_0}}\right) + 63\,Si + 42\sqrt{N_{sol}}$$

$$TS = \sqrt[3]{f_\alpha}\left(246 + 1140\sqrt{N_{sol}} + \frac{18.2}{\sqrt{d}}\right) + \left(1 - \sqrt[3]{f_\alpha}\right)\left(720 + \frac{3.5}{\sqrt{S_0}}\right) + 97\,Si$$

$$ITT = f_\alpha\left(-46 - \frac{11.5}{\sqrt{d}}\right) + \left(1 - f_\alpha\right)\left[-335 + \frac{5.6}{\sqrt{S_0}} - \frac{13.3}{\sqrt{p}} + 3.48\times10^6\,t\right] + 48.7\,Si + 762\sqrt{N_{sol}}$$

Notation:

YS: Yield Strength at 0.2% Real Strain [MPa]
TS: Tensile Strength [MPa]
ITT: Impact Transition Temperature for 50% Tough Fracture [°C]
f: Volume Fraction of Ferrite
d: Ferrite Grain Size [mm]
Alloy Content: [weight %]
$\mathbf{N_{sol}}$: Solubilized (Free) Nitrogen [%]
$\mathbf{S_0}$: Pearlite Lamelar Spacing [mm]
p: Pearlite Colony Size [mm]
t: Pearlitic Carbide Lamellar Thickness [mm]

Sources:

- GLADMAN, T. e outros. *Some Aspects of the Structure-Property Relationships in High Carbon Ferrite-Pearlite Steels*. **Journal of the Iron and Steel Institute**, 210, Dec. 1972, 916-930.

- PICKERING, F.B. *Some Aspects of the Relationships between the Mechanical Properties of Steels and their Microstructures*. **TISCO**. Silver Jubilee Volume, Jan-Oct 1980, 105-132.

. Microalloyed Steels (Hodgson)

$$YS = 62.6 + 26.1\,Mn + 60.2\,Si + 759.0\,P + 212.9\,Cu + 3286.0\,N_{sol} + \frac{19.7}{\sqrt{d}} + \Delta\sigma_{ppt}$$

$$TS = 164.9 + 634.7\,C + 53.6\,Mn + 99.7\,Si + 651.9\,P + 472.6\,Ni + 3339.4\,N_{sol} + \frac{11.0}{\sqrt{d}} + \Delta\sigma_{ppt}$$

$$\Delta\sigma_{ppt}^{Nb} = 2500\,Nb$$

$$\Delta\sigma_{ppt}^{V} = 57\log CR + 700\,V + 7800\,N_{sol} + 19$$

Notation:

YS: Yield Strength at 0.2% Real Strain [MPa]
TS: Tensile Strength [MPa]
Alloy Content: [weight %]
d: Grain Size [mm]
$\Delta\sigma_{ppt}^{Nb}$: Precipitation Strengthening [MPa], only for steels with Nb [MPa]
$\Delta\sigma_{ppt}^{V}$: Precipitation Strengthening [MPa], only for steels with V [MPa]
CR: Cooling Rate [°C/s]

Source: HODGSON, P.D. & GIBBS, R.K. *A Mathematical Model to Predict the Mechanical Properties of Hot Rolled C-Mn and Microalloyed Steels*. **ISIJ International**, 32:12, December 1992, 1329-1338.

. Microalloyed Steels (Pickering)

$$YS = \sigma_0 + 37\ Mn + 83\ Si + 2918\ N_{sol} + \frac{15.1}{\sqrt{d}} + \Delta\sigma_{ppt}$$

Notation:

YS: Yield Strength at 0.2% Real Strain [MPa]
σ_0: Friction Stress [MPa]
Alloy Content: [weight %]
d: Grain Size [mm]
$\Delta\sigma_{ppt}$: Precipitation Strengthening [MPa], for steels with Nb, Ti and/or V, defined by the formula below [MPa].

Observations:

- The Friction Stress **σ_0** value depends on the previous treatment of the material and can be found in the table below:

Condition	**σ_0 [MPa]**
Mean	70
Air Cooled	88
Overaged	62

- The effect of solid solution strengthening from another alloy elements dissolved in ferrite can be included in this equation, using the following linear coefficients:

Element	**MPa/weight %**
Ni	33
Cr	-30

P	680
Cu	38
Mo	11
Sn	120
C	5000
N	5000

- The precipitation strengthening contribution is calculated according to the Ashby-Orowan model.

$$\Delta\sigma_{ppt} = \frac{5.9\sqrt{f}}{\bar{x}} \ln\left(\frac{\bar{x}}{2.5\times10^{-4}}\right)$$

Notation:

$\Delta\sigma_{ppt}$: Precipitation Strengthening According to the Ashby-Orowan Model [MPa]
f: Volume Fraction of the Precipitate
x: Mean Planar Intercept Diameter of the Precipitate [μm]

Observations:

- Relationship adequate for the calculation of the precipitation strengthening of quench-aged carbides and precipitate carbonitrides in Nb, V and Ti steels.
- $\Delta\sigma_{ppt}$ can be calculated using a more simplified approach, multiplying the total content of the precipitating alloy by the factor **B** shown in the table below:

Alloy and Precipitate	**B_{min} [MPa/weight %]**	**B_{max} [MPa/weight %]**	**Alloy Range [weight %]**
V as V_4C_3	500	1000	0,00 ~ 0,15
V as VN	1500	3000	0,00 ~ 0,06
Nb as Nb(CN)	1500	3000	0,00 ~ 0,05
Ti as TiC	1500	3000	0,03 ~ 0,18

Source: PICKERING, F.B. *Some Aspects of the Relationships between the Mechanical Properties of Steels and their Microstructures.* **TISCO**. Silver Jubilee Volume, Jan-Oct 1980, 105-132.

. Microalloyed Nb, NbMo and TiMo Ferritic and Bainitic Steels (Isasti)

$$\sigma_y = \sigma_0 + \sigma_{ss} + \sigma_{gs} + \sigma_\rho + \sigma_{MA} + \sigma_{ppt}$$

$$\sigma_{ss} = \sigma_0 + 32.3\,Mn + 83.2\,Si + 11\,Mo + 354\sqrt{N_{sol}}$$

$$\sigma_{gs} = 1.05\,\alpha\,M\,\mu\sqrt{b}\left[\sum_{2\le\theta_i\le15^\circ} f_i\sqrt{\theta_i} + \sqrt{\frac{\pi}{10}}\sum_{\theta_i>15^\circ} f_i\right]\frac{1}{\sqrt{D_{2^\circ}}}$$

$$\sigma_\rho = \alpha\,M\,\mu\,b\sqrt{\rho},\ \text{where}\ \rho = \frac{2\,\vartheta}{u\,b}$$

$$\sigma_{MA} = 900\,f_{MA}$$

$$\sigma_{ppt} = 10.8\,\frac{\sqrt{f_v}}{x}\,ln\left(\frac{x}{6.125\,x\,10^{-4}}\right)$$

$$DBTT = -11\,Mn + 42\,Si + 700\sqrt{N_{sol}} + 15\sqrt[3]{f_{Pearl}+f_{MA}} + 0.26\,\Delta\sigma_y + \frac{14}{\sqrt{D_{15^\circ}}} + 63\sqrt{\frac{D_{c20\%}}{D_{15^\circ}}} + 18\sqrt{D_{MA}} - 42$$

Notation:

σ_0: Lattice Friction Stress (equal to 53.9 MPa)
σ_y: Yield Strength [MPa]
$\Delta\sigma_{ss}$: Contribution Due to Solid Solution [MPa]
Alloy Content: [weight %]
σ_{gs}: Contribution Due to Grain Size [MPa]
α: Constant (equal to 0.3 for $\Delta\sigma_{gs}$)
M: Average Taylor Factor (equal to 3)
μ: Shear Modulus (equal to 8 x 10^4 MPa for $\Delta\sigma_{gs}$)
b: Magnitude of the Burger's Factor (equal to 2.5 x 10^{-7}) [mm]
θ_i: Mean Misorientation Angle of Grain Boundary in the interval **i** [rad]
f_i: Relative frequency in the interval **i**
D_{2°: Mean Unit Size for the 2° Misorientation Criterion [μm]
σ_ρ: Contribution Due to Dislocation Density [MPa]
αM: Constant (equal to 0.38 for $\Delta\sigma_\rho$ in the case of body-centered cubic metals)
ϑ: Kernel Average Misorientation (measured by EBSD analysis) [°]
u: Unit Length (equal to 1.86 times the step size adopted in the EBSD analysis) [μm]
b: Burgers Vector
σ_{MA}: Contribution Due to Martensite-Austenite Constituent [MPa]
f_{MA}: Volume Fraction of Martensite-Austenite Constituent [%]
σ_{ppt}: Contribution Due to Precipitation [MPa]
f_v: Volume Fraction of Precipitates
x: Mean Planar Intecept Diameter of the Precipitates [μm]
f_{Pearl}: Volume Fraction of Pearlite [%]
$\Delta\sigma_y$: Sum of σ_ρ + σ_{ppt}
D_{15°: Mean Unit Size for the 15° Misorientation Criterion [μm]
$D_{c20\%}$: Cutoff grain size at 80% area fraction in a grain-size distribution histogram [μm]
D_{MA}: Grain Size of the Martensite-Austenite Constituent [μm]

Sources: ISASTI, N. et alii: *Microstructural Features Controlling Mechanical Properties in Nb-Mo Microalloyed Steels. Part I: Yield Strength.* **Metallurgical and Materials Transactions**, 45A:10, October 2014, 4960-4971.

ISASTI, N. et alii: *Microstructural Features Controlling Mechanical Properties in Nb-Mo Microalloyed Steels. Part II: Impact Toughness*. **Metallurgical and Materials Transactions**, 45A:10, October 2014, 4972-4982.

. Microalloyed VTiN Steels Processed by *Recrystallization Controlled Rolling*

$$YS = 41.4 + 575.20\, C_{eq} + (27401\, N_{ef} - 2)\, \sqrt{V} + \frac{419.5}{\sqrt{h_f}}$$

$$TS = 74.1 + 985.1\, C_{eq} + (31125\, N_{ef} - 39)\, \sqrt{V} + \frac{181.5}{\sqrt{h_f}}$$

Notation:

YS: Yield Strength at 0.2% Real Strain [MPa]
TS: Tensile Strength [MPa]
Alloy Content: [weight %]
$\mathbf{h_f}$: Plate Thickness [mm]

$$C_{eq} = C + \frac{Mn}{6} + \frac{Cr + Mo}{5} + \frac{Ni + Cu}{15}$$

$$N_{ef} = N_{tot} - \frac{Ti}{3.42}$$

Observations:

- Formula Derived for Steels with Al Content over 0.010% and Si Content between 0.25 and 0.35%.
- Precision of the Formulas: ± 40 MPa.

Source: MITCHELL, P.S. et alii.: *Effect of Vanadium on Mechanical Properties and Weldability of Structural Steels.* In: **Low Carbon Steels for the 90's**. Proceedings. American Society for Metals/The Metallurgical Society, Pittsburgh, Oct. 1993.

. Structural Steels: Ductile-Brittle Transition Temperature (Hannula 2015)

$$DBTT\ 34\ J/cm^2 = 42 - \frac{1038}{\sqrt{d_{90}}} + \left(0.341 - \frac{0.047}{\sqrt{d_{90}}}\right) YS$$

Notation:

DBTT 34 J/cm²: Ductile-Brittle Transition Temperature for Charpy Specific Energy of 34 J/cm² [°C]
d_{90}: 90th Percentile of Grain Size (Equivalent Circle Diameter) Distribution [µm]
YS: 0.2% Proof Stress [MPa]

Observations:

- Useful range: **DBTT 34 J/cm²**, -50~-120°C; **d_{90}**, 7~26 µm; **YS**, 910~1070 MPa.

Source: HANNULA, J. et alii.: *Effect of Niobium and Boron on the Strength and Toughness of Abrasive Wear Resistant Direct-Quenched Low-Carbon Steel.* International Symposium on Wear Resistant Alloys for the Mining and Processing Industry, **Proceedings**. CBMM, Campinas, 2015.

. Structural Steels: Impact Transition Temperature(Mintz 1979)

$$ITT\ 27J = 173\sqrt{t} - \frac{8.3}{\sqrt{d}} + 0.37\,\Delta\sigma_{ppt}$$

$$ITT\ 54J = 192\sqrt{t} - \frac{10.1}{\sqrt{d}} + \frac{\Delta\sigma_{ppt}}{2}$$

$$50\%\ FATT = 131\sqrt{t} - \frac{12.7}{\sqrt{d}} + 0.45\,\Delta\sigma_{ppt} + 46$$

Notation:

ITT 27 J: Impact Transition Temperature for Charpy Energy of 27 J [°C]
ITT 54 J: Impact Transition Temperature for Charpy Energy of 54 J [°C]
50% FATT: 50% Fibrous Fracture Appearance Transition Temperature [°C]
t: Carbide Thickness as Measured by Scanning Electron Microscopy [μm]
d: Grain Size [μm]
$\Delta\sigma_{ppt}$: Precipitation Hardening [MPa]

Observations:

- Useful range: **C**, 0.11~0.20%; **Mn**, 0.63~1.56%; **Si**, 0.02~0.49%; **N_{total}**, 0.003~0.021%; **Nb_{max}**, 0.071%; **V_{max}**, 0.20%; **Ti_{max}**, 0.16%; **Al_{sol}**, up to 0.12%; **t**, 0.16~0.72 μm; **d**, 4.9~38.5 μm; **$\Delta\sigma_{ppt}$,** up to 225 MPa.

Source: MINTZ, B. et alii.: *Influence of Carbide Thickness on Impact Transition Temperature of Ferritic Steels.* **Metals Technology**, July 1979, 252-260.

. Structural Steels: Impact Transition Temperature(Mintz 1994)

$$ITT\ 27J = 80.1 - \frac{7.41}{\sqrt{d}} + 1.4\ p - 57.2\ Si + 1224\ S - 1360\ P - 3.7\sqrt{CR} - 57.8\ Mn$$

$$ITT\ 54J = 84.8 - \frac{5.65}{\sqrt{d}} + 1.67\ p - 53.1\ Si + 1490\ S - 1379\ P - 4.97\sqrt{CR} - 70.1\ Mn$$

Notation:

ITT 27 J: Impact Transition Temperature for Charpy Energy of 27 J [°C] – $r^2 = 0.950$; s = 8.08

ITT 54 J: Impact Transition Temperature for Charpy Energy of 54 J [°C] – $r^2 = 0.955$; s = 7.80
d: Grain Size [μm]
p: Pearlite Fraction [%]
CR: Cooling Rate [°C/min]

Observations:

- Useful range: **C**, 0.067~0.220%; **Mn**, 0.56~1.51%; **Si**, 0.02~0.43%; $\mathbf{N_{total}}$, 0.0025~0.010%; $\mathbf{Nb_{max}}$, 0.030%; $\mathbf{Al_{max}}$, 0.041%.
- Recommended only for normalised steels with ferrite-pearlite microstructure.

Source: MINTZ, B. et alii.: *Structure-Property Relationships in Ferrite-Pearlite Steels.* **Ironmaking and Steelmaking**, 21:3, 1994, 215-222.

. Non-Oriented Si Electrical Steels

$$YS = 34.3 + \frac{22.0}{\sqrt{d}} + 258\,P + 34.2\,Mn + 52.8\,Si$$

$$TS = 183 + \frac{11.2}{\sqrt{d}} + 506\,P + 48.7\,Mn + 109\,Si + 48.8\,Al + 2450\,B$$

$$YR = 0.424 + \frac{0.0412}{\sqrt{d}} - 0.078\,Si - 0.170\,Al$$

Notation:

YS: Lower Yield Strength [MPa]
TS: Tensile Strength [MPa]
YR: Yield Ratio
d: Ferrite Grain Size [mm]

Alloy Content: [weight %]

Observations:

- These equations are valid under the following conditions: **ULC Steel**; **Mn**: 0.075~0.578%; **P** < 0.109%; **S**: 0.003~0.004%; **Si** < 0.34%; **Al**: < 0.432%; **N**: 0.0014~0.0020%; **B** < 0.0030%.
- Cold rolled steel was box annealed at 700°C; the time of treatment, including heating of the samples, was equal to 32 hours, being followed by furnace cooling.

Source: PINOY, L et alii. *Influence of Composition and Hot Rolling Parameters on the Magnetic and Mechanical Properties of Fully Processed Non-Oriented Low-Si Electrical Steels.* **J. Phys. IV France**, 8, 1998, Pr2-487/Pr2-490.

$$P_T = 0.658 - 0.474\,Si - 2.311\,Al - 25.99\,O + 12.51\,C + 123.7\,S_{init} + 130.2\,\Delta S - 137.5\,N + 5.266\,h$$

Notation:

P_T: Core Loss [W/kg]
Alloy Content: [weight %]
h: Thickness [mm]

Observations:

- This equation ise valid under the following conditions: **C**: 0.002~0.040%; **S**: 0.004~0.015%; **N**: 0.003~0.007%.
- Negative effects of O and N are in direct contradiction with specific experimental results.
- Adjusted Squared Multiple Correlation: 0.823; Residual Mean Square: 0.082 W^2/kg^2

$$P_T = 4.29 + 66.4\,C + 0.0282\,GBI + 16.2\,\frac{h^2}{\rho\,r}$$

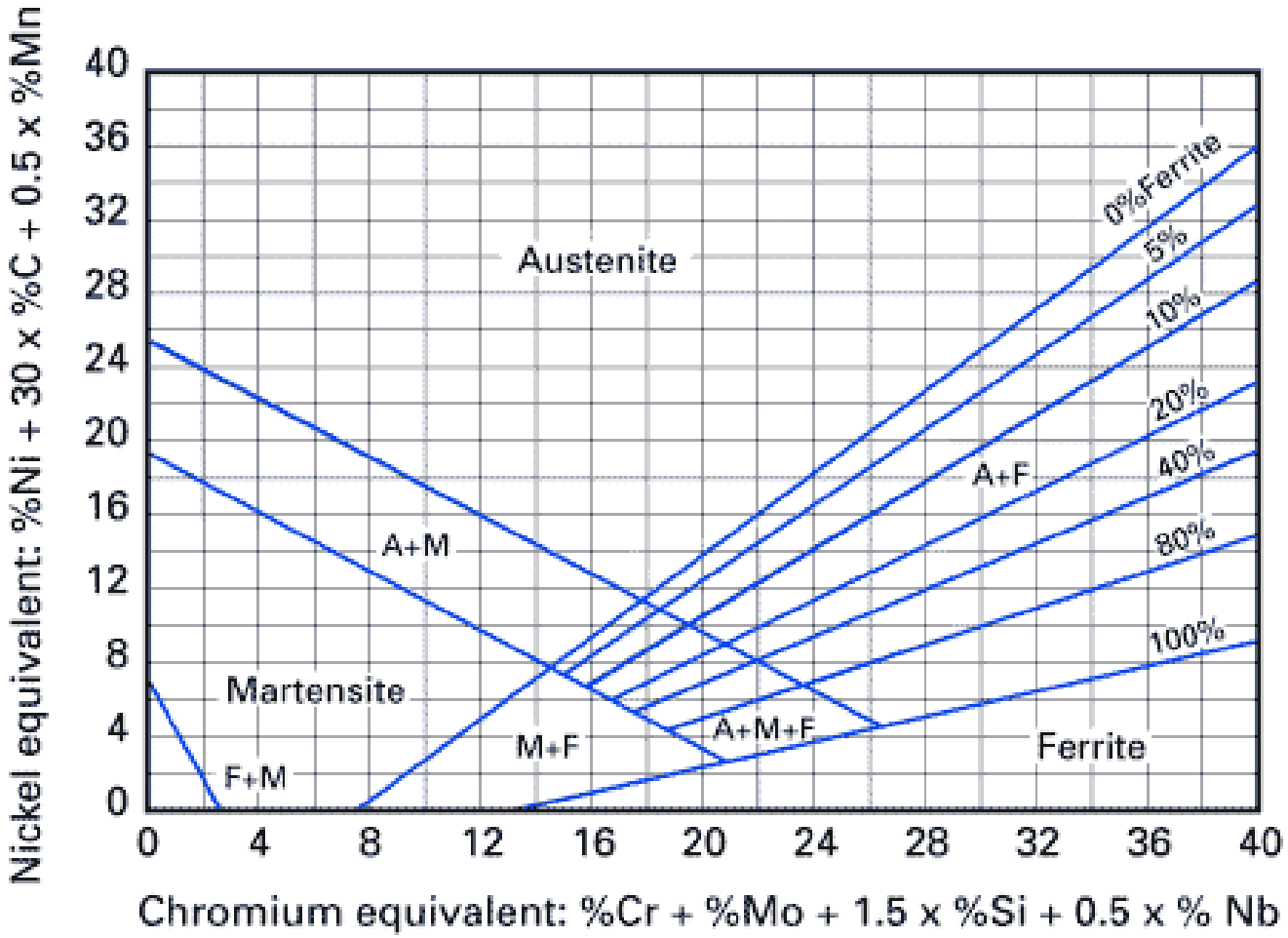
Notation:

P_T: Total Core Loss at 15 KG [W/kg]

Alloy Content: [weight %]
GBI: Number of Grain Boundary Intercepts per mm
h: Thickness [mm]
ρ: Density [g/mm³]
r: Resistivity [μΩ.mm]

Source: LYUDKOVSKY, G. et alii. *Non-Oriented Electrical Steels.* **Journal of Metals**, January 1986, 18-26.

- **Schaeffler Diagram**



Source: Air Products Web Site
(http://www.airproducts.com/maxx/software/UK/WeldingFaultFinder/wff22413.html).

- **Shear Modulus of Steel and its Phases**

. Ferrite

$$\mu = 64000\left[1 - \frac{(T - 300)}{2235}\right] \qquad (-273°C < T < 300°C)$$

$$\mu = 64000\left[1 - \frac{(T - 300)}{2235}\right] - 0.032\,(T - 573)^2 \qquad (300°C \le T < 700°C)$$

$$\mu = 64000\left[1 - \frac{(T - 300)}{2235}\right] - 0.032\,(T - 573)^2 - 0.024\,(T - 923)^2 \qquad (700°C \le T < 770°C)$$

$$\mu = 69200\left[1 - \frac{(T - 300)}{1382}\right] \qquad (770°C \le T < 911°C)$$

. Austenite

$$\mu = 81000\left[1 - \frac{(T - 300)}{1989}\right] \qquad (911°C \le T < 1392°C)$$

. Delta Ferrite

$$\mu = 39000\left[1 - \frac{(T - 300)}{2514}\right] \qquad (1392°C \le T < 1537°C)$$

Notation:

μ: Shear Modulus [MPa]
T: Temperature [K]

Source: FROST, H.J. & ASHBY, M.F.: *Pure Iron and Ferrous Alloys.* In: **Deformation-Mechanism Maps, The Plasticity and Creep of Metals and Ceramics**. Pergamon Press, Cambridge, 1982.

- **Sheet and Plate Cutting Force and Work**

. Mesquita

$$K_c = 0.88\,TS$$

$$F_c = t\,P\,K_c$$

Notation:

K_c: Cutting Specific Pressure or Shear Stress [MPa]
TS: Tensile Strength [MPa]
F_c: Cutting Force [N]
t: Thickness [mm]
P: Cutting Perimeter [mm]

Source: MESQUITA, E.L.A. *Conformação dos Aços Inoxidáveis.* **Manual da Acesita**. Dezembro 1997, 39 p.

. Tschaetsch

$$F = W\,t\,\tau_B$$

$$W = \frac{a\,F\,t}{1000}$$

$$P = \frac{F\,v}{\eta}$$

Notation:

F: Cutting Force [N]

W: Width [mm]
t: Thickness [mm]
W: Cutting Work [N.mm]
a: Mean Force/Maximum Force Ratio (≈ 0.6 for shearing)
P: Cutting Power [W]
v: Shear Speed [m/s]
η: Machine Efficiency (≈ 0.7)
τ_B: Shear Stress [MPa], as defined by the table or formulas described in the observation below.

Observations:

- The value of **τ_B** can be calculated from the following equations, where C is the carbon weight content of steel:

. Hot rolled or annealed steel (soft) – $r^2 = 0.992$, Standard Error of Deviation = 13 MPa:

$$\tau_B = 223 + 550\,C$$

. Cold rolled steel (hard) – $r^2 = 0.988$, Standard Error of Deviation = 9 MPa:

$$\tau_B = 249 + 786\,C$$

- These equations were fitted using the **τ_B** data available below, expressed in N/mm^2:

Steel	**C**	**Mn**	**Si**	**Soft**	**Hard**
St 12	0.10 max	0.50 max	-	240	300
St 13	0.10 max	0.50 max	-	240	300
St 14	0.08 max	0.40 max	-	250	320

St 37	0.20 max	1.25 max	0.25 max	310	-
St 42	0.25 max	1.25 max	0.25 max	400	-
C10	0.08-0.13	0.30-0.60	0.30 max	280	340
C20	0.18-0.23	0.30-0.70	0.30 max	320	380
C30	0.27-0.34	0.50-0.80	0.10-0.40	400	500
C60	0.57-0.65	0.60-0.90	0.15-0.35	550	720

Source: TSCHAETSCH, H. *Shearing.* **Metal Forming Practice - Processes, Machines, Tools**; Springer-Verlag, Berlin, 2006, 218-40.

- **Solidus Temperature of Steels**

. Qian

$$T_{Sol} = a - b\,C_e$$

$$C_e = \frac{80.5\,C + 33.5\,(S + P) + 3.75\,Mn + 17.8\,Si + 3.4\,Co + 3.8\,Al + 1.5\,Cr + 3\,Ni}{80.5}$$

Notation:

T_{Sol}: Steel Solidus Temperature [°C]
a: Constant equal to
. 1493, for $0.1 \le C_e \le 0.2\%$;
. 1534, for $C_e < 0.1\%$ or $C_e > 0.2\%$;
b: Constant equal to
. 0, for $0.1 \le C_e \le 0.2\%$;
. 410, for $C_e < 0.1\%$;
. 184, for $C_e > 0.2\%$;
Alloy Content: [weight %]

Source: QIAN, H. et alii. *Effect of the Non-Equilibrium Solidus Temperature on Length of Soft Reduction Zone*. **Advanced Materials Research**, 476-478, 2012, 139-143.

. Takeuchi

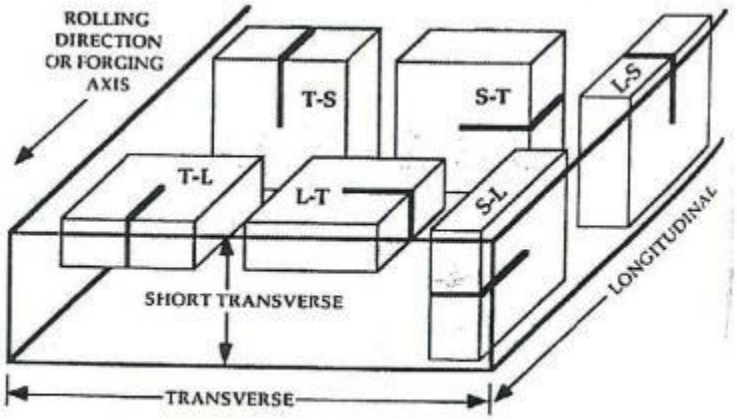
$$T_{Sol} = 1536 - \left[415{,}5\,C + 12{,}3\,Si + 6{,}8\,Mn + 124{,}5\,P + 183{,}9\,S + 4{,}3\,Ni + 1{,}4\,Cr + 4{,}1\,Al\right]$$

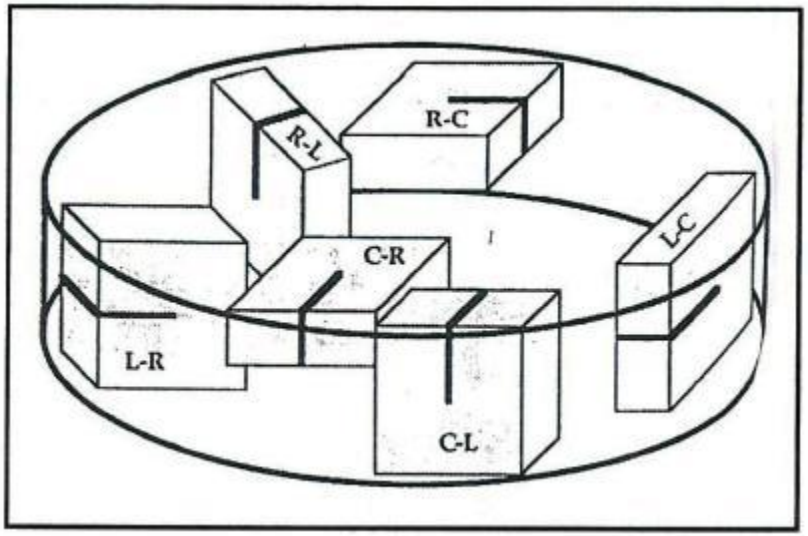
Notation:

T_{Sol}: Steel Solidus Temperature [°C]
Alloy Content: [weight %]

Source: TAKEUCHI, E. & BRIMACOMBE, J.K. *Effect of Oscillation-Mark Formation on the Surface Quality of Continuously Cast Steel Slabs*. **Metallurgical Transactions B**, 16B, 9, 1985, 605-625.

- **Specimen Orientation for Mechanical Testing**





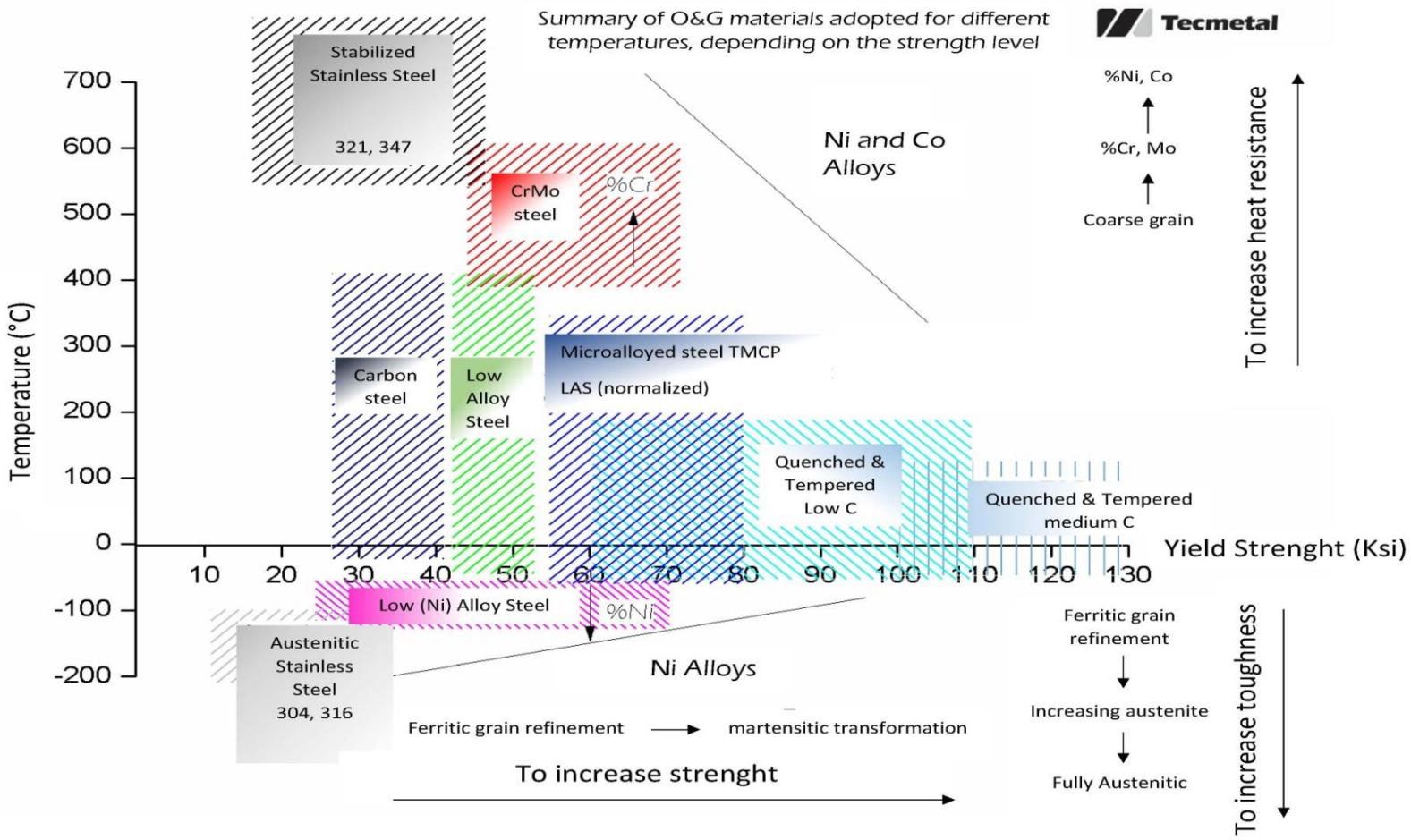
Notes:

- The first letter denotes the direction of the applied main tensile stress.

- The second letter denotes the direction of crack propagation.

Source: ASTM Standard E399-06. **Standard Test Method for Linear-Elastic Plane-Strain Fracture Toughness K_{IC} of Metallic Materials**. ASTM International, West Conshohocken, 2006, 32 p.

- **Steel Properties Map**



Source: www.tecmetal.com.br

- **Thermal Properties of Steel**

. BISRA

T	k			
	1008	**1023**	**1040**	**1524**
75	481.39	485.58	485,58	477,20
125	502.32	506.51	502.32	493.95
175	523.25	519.16	514.88	510.69
225	544.18	531.62	527.44	527.44
275	556.74	556.74	548.37	544.18
325	569.30	537.48	569.30	565.11
375	594.41	598.60	586.04	590.23
425	623.71	623.71	611.16	615,34
475	661.39	661.39	648.83	648.83
525	694.88	703.25	690.69	694.88
575	740.92	749.29	707.43	740.92
625	753.48	78697	732.55	753.48
675	858.13	845.57	770.22	837.20
725	1138.59	1431.61	1582.31	1448.36
775	958.59	950.22	602.78	820.46
825	866.50	736.74	611.16	573.48
875	648.83	648.83	615.34	581.85
925	648.83	648.83	623.71	590.23
975	657.20	648.83	623.71	598.60
1025	657.20	648.83	632.09	606.97

1075	661.39	648.83	632.09	615.34
1125	661.39	657.20	640.46	623.71
1175	665.57	665.57	653.02	632.09
1225	665.57	678.13	669.76	636.27
1275	665.57	686.50	686.50	644.64

T	**H**			
	1008	**1023**	**1040**	**1524**
0	0	0	0	0
25	11.459	11.564	11.613	11.407
75	35.005	35.319	35.467	34.848
125	59.598	60.121	60.164	59.127
175	85.237	85.761	85.594	84.243
225	111.923	112.028	111.652	110.196
275	139.446	139.237	138.547	136.987
325	167.597	167.492	167.489	164,719
375	196.960	196.794	195,372	193.603
425	227.143	227.352	225.302	223.742
475	259.270	259.480	256.802	255.346
525	293.177	293.596	290.290	288.939
575	329.072	329.909	325.243	324.834
625	366.432	368.316	361.242	362.194
675	406.722	409.129	398.812	401.961
725	456.640	466.059	457.625	459.100
775	509.070	525.605	512.252	515.820

825	554.697	567.779	542.601	550.668
875	592.581	602.418	573.263	579.552
925	625.022	634.859	604.240	608.854
975	657.673	667.307	635.425	638.574
1025	690.533	699.742	666.820	668.714
1075	723.498	732.184	698.425	699.271
1125	756.567	764.835	730.238	730.248
1175	789.741	797.904	762.575	761.643
1225	823.020	831.497	795.644	793.352
1275	856.299	865.612	829.551	825.375

Notation:

k: Thermal Conductivity [W/(m.°C)]
c: Specific Heat Capacity [J/(kg.°C)]
H: Enthalpy [J/kg]
T: Temperature [°C]

Observations:

- Chemical composition of the steels [wt %]:

Steel	**C**	**Mn**	**Si**	**P**	**S**	**Cu**
1008	0.08	0.31	0.08	0.029	0.050	-
1023	0.23	0.64	0.11	0.034	0.034	0.13
1040	0.42	0.64	0.11	0.031	0.029	0.12
1524	0.23	1.51	0.12	0.037	0.038	0,11

Source: *Physical Constants of Some Commercial Steels at Elevated Temperatures*, BISRA/Butterworths Scientific Publications, London, 1953, 1-38.

. Chou

$k = 80.91 - 9.9269 \times 10^{-2}\ T + 4.613 \times 10^{-5}\ T^2$ (T ≤ Ar_3)

$k = 20.14 + 9.313 \times 10^{-3}\ T$ (T > Ar_3)

$$c = -4720.324 + 4.583364\,T + \frac{1.109483 \times 10^9}{T^2}$$ (800K < T < 1000K)

$c = -11501.07 + 12.476362\,T$ (1000K < T < 1042K)

$c = 34871.21 + 32.02658\,T$ (1042K < T < 1060K)

$$c = -10068.18 + 5.98686\,T + \frac{5.217657 \times 10^9}{T^2}$$ (1060K < T < 1184K)

$c = 429.8495 + 0.1497802\,T$ (1084K < T < 1665K)

Notation:

k: Thermal Conductivity [J/m.K.s]
c: Specific Heat Capacity [J/kg.K]
T: Temperature [K]

Source: SEREDYNSKI, F.: *Performance Analysis and Optimization of the Plate-Rolling Process.* In: **Mathematical Process Models in Iron and Steelmaking**. Proceedings. Iron and Steel Institute, Amsterdam, 1973.

. Krzyzanowski

$$c_p = 422.7 + 48.66 \exp\left(0.319 \times 10^{-5}\ T\right) \qquad (T \le 700°C)$$

$$c_p = 657.0 + 0.084 \left(\frac{T}{1000}\right)^{-24,6} \qquad (T > 700°C)$$

$$\lambda = 23.16 + 51.96 \exp\left(-2.02519 \times 10^{-3}\ T\right)$$

$$\rho = \frac{7850}{\left(1 + 0.004 \times 10^{-6}\ T^2\right)^3}$$

Notation:

c_p: Specific Heat [J/kg.°C]
T: Temperature [°C]
λ: Thermal Conductivity [J/m.°C.s]
ρ: Density [kg/m³]

Source: KRZYZANOWSKI, M. et alii.: *Finite Element Model of Steel oxide Failure During Tensile Testing Under Hot Rolling Conditions.* **Materials Science and Technology**, 15:10, October 1999, 1191-1198.

. Seredynski

$$k = -58.6 \times 10^{-3}\ T + 72.5 \quad (T < 810°C)$$

$$k = 10.75 \times 10^{-3}\ T + 16.8 \quad (T > 810°C)$$

$$D = 0.15 \times 10^{-7}\ T - 0.07825 \times 10^{-4} \quad (700°C < T < 875°C)$$

$$D = 0.02667 \times 10^{-7}\ T - 0.02966 \times 10^{-4} \quad (T > 875°C)$$

$$\varepsilon = \frac{T}{1000}\left(0.12491\frac{T}{1000} - 0.38012\right) + 1.0948$$

Notation:

k: Thermal Conductivity [J/m.°C.s]
D: Thermal Diffusivity [m²/s]
ε: Emissivity
T: Temperature [°C]

Observation:

- Formulas specific for BS En 3 or SAE 1021 steel: 0.17-0.23% C; 0.60-0.90% Mn

Source: SEREDYNSKI, F.: *Performance Analysis and Optimization of the Plate-Rolling Process.* In: **Mathematical Process Models in Iron and Steelmaking**. Proceedings. Iron and Steel Institute, Amsterdam, 1973.

. Touloukian

$$\varepsilon = \frac{0{,}85}{[1 + \exp(42.68 - 0{,}02682\,T_{sup})^{0{,}0115}]}$$

Notation:

ε: Emissivity
T_{sup}: Superficial Temperature [K]

Source: HARDIN, R.A. et alii.: *A Transient Simulation and Dynamic Spray Cooling Control Model for Continuous Steel Casting*. **Metallurgical and Materials Transactions B**, 34B:6, June 2003, 297-306.

- **Thermal Properties of Steel Scale**

. Krzyzanowski

$k = 1 + 7.833 \times 10^{-4}\ T$ (873K < T < 1473K)

$c = 674.969 + 0.297\ T - 4.367 \times 10^{-5}\ T$ (600°C < T < 1100°C)

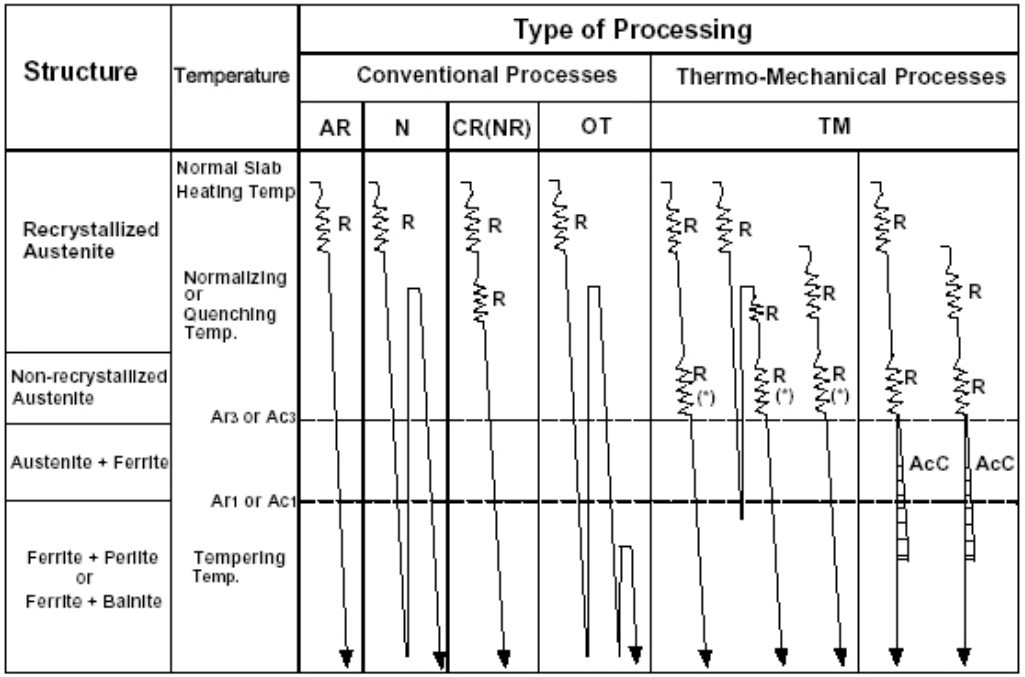
$E = 240\left[1 - 4.7 \times 10^{-4}\ (T - 25)\right]$

Notation:

k: Conductivity [W/m.K]
c: Specific Heat Capacity [J/kg.°C]
E: Young's Modulus [GPa]
T: Temperature [°C]

Source: KRZYZANOWSKI, M. et alii.: *Finite Element Model of Steel oxide Failure During Tensile Testing Under Hot Rolling Conditions.* **Materials Science and Technology**, 15:10, October 1999, 1191-1198.

- **Thermomechanical Processing of Steel**



Notes:

AR:	As Rolled
N:	Normalizing
CR(NR):	Controlled Rolling (Normalizing Rolling)
QT:	Quenching and Tempering
TM:	Thermo-Mechanical Rolling (Thermo-Mechanical Controlled Process)
R:	Reduction
(*):	Sometimes rolling in the dual-phase temperature region of austenite and ferrite
AcC:	Accelerated Cooling

Source: *Requirements Concerning Materials and Welding*. **IACS – International Association of Classification Societies Requirement 1975**, Revision 2, 2004, 228 p.

- **Time-Temperature Equivalency Parameters for Heat Treating**

. Austenitizing (Anisothermal)

In this case the Austenitization Time-Temperature Equivalence Parameter in Terms of Grain Size, $\mathbf{P_a}$, is the period of heating/cooling time between $\mathbf{T_{max}}$ and $\mathbf{T_{min}}$, where

$\mathbf{T_{max}}$: Maximum Temperature during the Austenitizing Treatment [°C];

$\mathbf{T_{min}}$: Temperature Calculated According to the Following Equation [°C]:

$$T_{min} = T_{max} - \frac{R\,T_{max}^2}{\Delta H_a}$$

Notation:

R: Molar Gas Constant, 8.314 $JK^{-1}mol^{-1}$

$\mathbf{\Delta H_a}$: Activation Energy of Austenitic Grain Coarsening, 460 $kJmol^{-1}$ for low alloy steels

Source: BARRALIS, J. & MAEDER, G. *Métallurgie Tome I: Métallurgie Physique.* **Collection Scientifique ENSAM**, 1982, 270 p.

. Austenitizing (Isothermal)

$$P_a = \frac{1}{\left[\frac{1}{T_a} - \frac{2{,}3\,R}{\Delta H_a}\log t_a\right]}$$

Notation:

$\mathbf{P_a}$: Austenitization Time-Temperature Equivalence Parameter in Terms of Grain Size [K]

T_a: Austenitization Temperature [K]
R: Molar Gas Constant, 8.314 $JK^{-1}mol^{-1}$
t_a: Soaking time under **T_a**
ΔH_a: Activation Energy of Austenitic Grain Coarsening, 460 $kJmol^{-1}$ for low alloy steels

Source: BARRALIS, J. & MAEDER, G. *Métallurgie Tome I: Métallurgie Physique.* **Collection Scientifique ENSAM**, 1982, 270 p.

. Microstructural Banding Homogeneization

$$t_{0.05} = \frac{0.3\, l^2}{D_0\, e^{\frac{-Q}{RT}}}$$

Notation:
$t_{0.05}$: Treatment Time Necessary to Achieve 5% Microstructure Banding [min]
l: Mean Spacing between Bands [mm]
D_o: Diffusion Constant for the Alloy Element being Considered [cm^2/s]:
- P: 0.01 cm^2/s
- Mn: 0.16 cm^2/s

Q: Activation Energy for the Alloy Element Being Considered [cal/mol]:
- P: 43700 cal/mol
- Mn: 62500 cal/mol

R: Molar Gas Constant, 1.987 $JK^{-1}mol^{-1}$
T: Austenitization Temperature [K]

Source: YIMING, X. et alii.: *A Metallographic Investigation of Banding and Diffusion of Phosphorus in Steels.* **Microstructural Science**, 20, 1993, 457-470.

. Tempering (Anisothermal)

$$\boxed{P = T\,(c + log\,\tau)}$$

$$\boxed{\tau = t + \frac{T}{2.303\,k_1\,(c - \log k_1)} + \frac{T}{2.303\,k_2\,(c - \log k_2)}}$$

Notation:

P: Hollomon-Jaffe Parameter [K]
T: Tempering Temperature [K]
c: Constant Characteristic of the Steel Being Tempered
t: Soaking Time Under **T** [h]
$\mathbf{k_1}$: Heating Rate [K/hour]
$\mathbf{k_2}$: Cooling Rate [K/hour]

Observations:

- See the section **Tempering (Isothermal)** for the values of constant **c**.

Sources:

- HOLLOMON, J.H. & JAFFE, L.D. *Time-Temperature Relations in Tempering Steel*. **Transactions of the AIME**, 162, 1945, 223-249.

. Tempering (Isothermal)

$$\boxed{P = T\,(c + \log t)}$$

$$\boxed{c = 21.53 - 5.8\,C}$$

Notation:

P: Hollomon-Jaffe Parameter [K]
T: Tempering Temperature [K]
c: Constant Characteristic of the Steel Being Tempered
t: Soaking time under **T** [h]
C: Carbon Content [wt%]

Observations:

- Other values for the constant **c** were proposed by several authors for carbon, microalloyed and low alloy steels:
  . 18 (Grange & Baughman)
  . 20 (Larson & Miller, Irvine, Thelning)

- This expression was also deduced by Larson & Miller, which applied it to the study of metal creep. In that case **c** is equal to 20 and **P** is divided by 1000. Such relationship was also used for the study of hydrogen resistance and HAZ hardness of steels.

Sources:

- HOLLOMON, J.H. & JAFFE, L.D. *Time-Temperature Relations in Tempering Steel*. **Transactions of the AIME**, 162, 1945, 223-249.

- LARSON, F.R. & MILLER, J. *A Time-Temperature Relationship for Rupture and Creep Stresses*. **Transactions of the American Society of Mechanical Engineers**, 74, 1952, 765-775.

- GRANGE, R.A. & BAUGHMAN, R.W. *Hardness of Tempered Martensite in Carbon and Low Alloy Steels*. **Transactions of the American Society for Metals**, 68, 1956, 165-197.

- THELNING, K.E.: *Steel and its Heat Treatment – Bofors Handbook*. Butterworths, London, 1981, 570 p.

- IRVINE, K.J. et alii. *Grain-Refined C-Mn Steels*. **Journal of the Iron and Steel Institute**, Feb. 1967, 161-182.

- **Welding Effects**

. Weld Interface Cracking Susceptibility during Flash Butt Welding

$$F_{eq} = (C - 0.03)\left[Si^2 + \left(\frac{Mn}{10}\right)^2 + (4\ Al)^2 + \left(\frac{3\ Cr}{2}\right)^2 \right]$$

Notation:

F_{eq}: Weld Interface Cracking Susceptibility during Flash Butt Welding (No Crack = Zero)
Alloy Content: [weight %]

Source: MIZUI, M. et alii.: *Application of High-Strength Steel Sheets to Automotive Wheels*. **Nippon Steel Technical Report**, 23, June 1984, 19-30.

. Tensile Strength after Flash Butt Welding

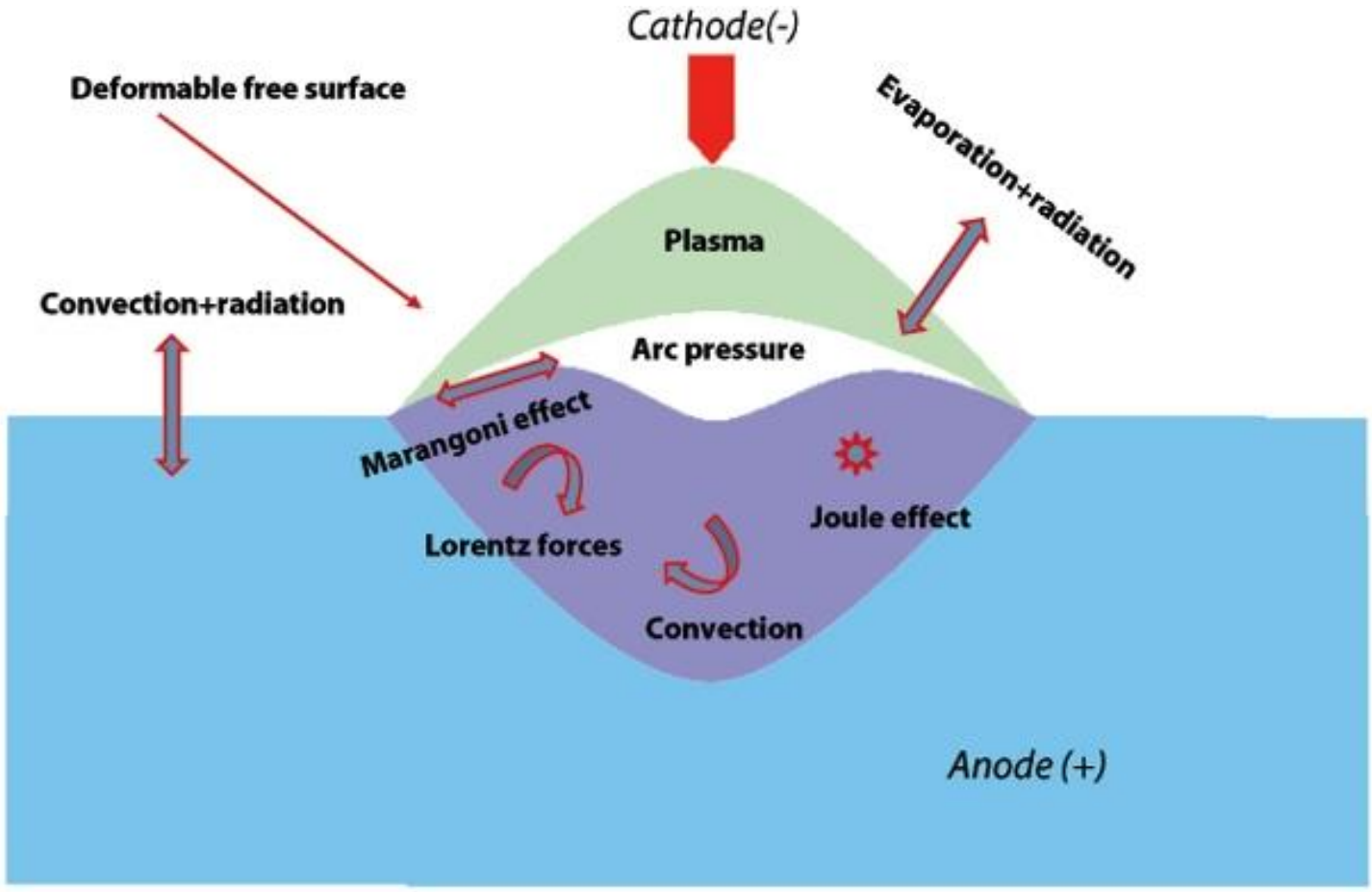
$$TS_{eq} = 52\left(C + \frac{Mn}{5} + \frac{Si}{7} + \frac{Cr}{9} + \frac{V}{2} \right) + 30$$

Notation:

TS_{eq}: Tensile Strength After Flash Butt Welding [kgf/mm²]
Alloy Content: [weight %]

Source: MIZUI, M. et alii.: *Application of High-Strength Steel Sheets to Automotive Wheels*. **Nippon Steel Technical Report**, 23, June 1984, 19-30.

- **Welding Pool Phenomena**



Source: ROGER, F. *Modeling Finds the Minimum Energy for the Best Weld*. Comsol News, 2010, p. 55-58.

- **Young Modulus**

. Definition

$$E = 2\,G\,(1+\upsilon)$$

Notation:

E: Young Modulus
G: Shear Modulus
υ: Poisson Ratio
. Elastic Range: 0.3
. Plastic Range: 0.5

Source: WILSON, A.D. *Guidelines for Fabricating and Processing Plate Steel*. Bethlehem-Lukens Plate Report, Burns Harbor, 2000, 97 p.

. Steel, High Temperature: Pietrzyk

$$E = \left[2.07 + 0.87438\left(\frac{T}{1000}\right) - 10.0906\left(\frac{T}{1000}\right)^2 + 14.48466\left(\frac{T}{1000}\right)^3 - 6.20767\left(\frac{T}{1000}\right)^4 \right] 10^5$$

Notation:

E: Young Modulus [MPa]
T: Temperature [°C]

Observation:

- Valid for steel. No information available about the range of valid temperatures.

Source: PICQUÉ, B. *Experimental Study and Numerical Simulation of Iron Oxide Scales Behavior in Hot Rolling*. **Doctor Thesis**, École de Mines de Paris, 2004, p. 243.

. Steel, High Temperature: Tselikov

$$E = 308250 + 42924\,C - 144000\,C^2 + 20525\,Si - 5289\,Mn - 12000\,P + 174000\,S - 225{,}6\,T + 0{,}01379\,T^2$$

Notation:

E: Young Modulus [kgf/cm²]
C: C content [weight %]
Mn: Mn content [weight %]
Si: Si content [weight %]
P: P content [weight %]
S: S content [weight %]
T: Temperature [°C]

Observation:

- Valid for carbon, alloy and stainless steels between 20 and 900°C.

Source: ROYZMAN, S.E. *Thermal Stresses in Slab Solidification*. **Asia Steel**, 1996, 158-162.

APPENDIXES

- **Greek Letters**

Upper Case	Lower Case	Name
Α	α	Alpha
Β	β	Beta
Γ	γ	Gamma
Δ	δ	Delta
Ε	ε	Epsilon
Ζ	ζ	Zeta
Η	η	Eta
Θ	θ	Theta
Ι	ι	Iota
Κ	κ	Kappa
Λ	λ	Lambda
Μ	μ	Mu

Upper Case	Lower Case	Name
Ν	ν	Nu
Ξ	ξ	Xi
Ο	ο	Omicron
Π	π	Pi
Ρ	ρ	Rho
Σ	σ	Sigma
Τ	τ	Tau
Υ	υ	Upsilon
Φ	ϕ	Phi
Χ	χ	Chi
Ψ	ψ	Psi
Ω	ω	Omega

- **Statistical Formulas - General**

Correlation Coefficient

$$r = \pm \sqrt{\frac{\Sigma (Y_{est} - \bar{Y})^2}{\Sigma (Y - \bar{Y})^2}}$$

Notation:

r: Correlation Coefficient
Y: Raw Data
$\mathbf{Y_{est}}$: Estimated Data Calculated by the Fitted Equation
$\bar{Y}$: Mean of the Raw Data

Source: SPIEGEL, M.R. **Estatística**, Editora McGraw-Hill do Brasil Ltda., São Paulo, 1976, 580 p.

Root Mean Square Deviation

$$\sigma = \sqrt{\frac{\Sigma (Y_{est} - Y_{real})^2}{n}}$$

Notation:

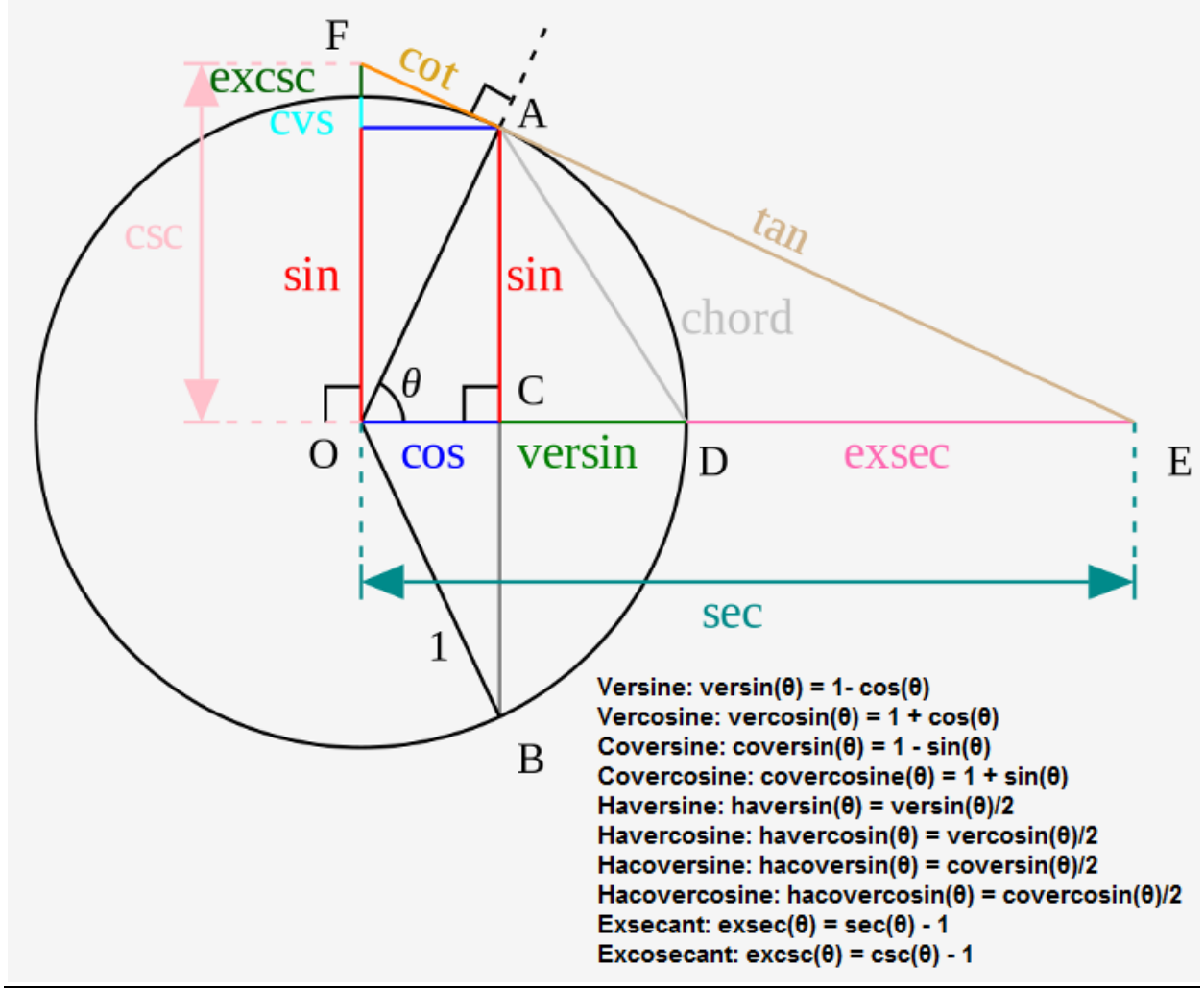
σ: Root Mean Square Deviation
$\mathbf{Y_{est}}$: Estimated Data Calculated by the Fitted Equation

$\mathbf{Y_{real}}$: Real Data
n: Number of Points of Data

Source: SPIEGEL, M.R. **Estatística**, Editora McGraw-Hill do Brasil Ltda., São Paulo, 1976, 580 p.

- **Trigonometry**

Trigonometrical Relations



- **Unit Conversions**

From	Multiply by	To
A	10^{-10}	m
bar	1.019716	kg/cm²
BTU	1058.201058	J
BTU	251.9958	cal
cal	4.184	J
F	5/9 (°F-32)	°C
ft	12	inch
ft	0.30485126	m
ft.lb	1.356	J ou N.m
ft.lb	0.324	cal
ft.lb	1.355748373	J
ft.lb/s	1.355380862	W
ft.lbf	1.355818	J or N.m
ft.lbf	0.1382	kg
ft²	92.90 x 10^{-3}	m²
ft³	0.02831685	m³
gallon	3.78541178	liters
HP	0.7456999	kW
HP	745.7121551	W
in	25.4	mm
in²	645.2	mm²
in³	16387.064	mm³
in-lb/in²	0.000175127	J/mm²
J	9.45 x 10^{-4}	BTU
J	0.2390	cal
J	0.7376	ft.lb

From	Multiply by	To
J	2.389 x 10^{-7}	th
J	1	W.s
J	2.777 x 10^{-9}	kWh
Kcal/m² h °C	1,163	W/m² °C
Kg	2.205	lb
kgf	9.80665	N
kgf.m	9.80665	J
kgf/mm²	9.80665	MPa
Kip	1000	lbf
kN	224.8	lbf
kN	0.102040816	t
kN/mm	5.71 x 10^{3}	lbf/ft
ksi	6.894757	MPa
ksi	1000	psi
ksi.√in	1.098901099	MPa.√m
kW	1.341022	HP
kW	0.860	th/h
kW.h	3.6 x 10^{6}	J
kW.h	3.412 x 10^{3}	BTU
kW.h	8.6 x 10^{5}	cal
lb	0.4535924	kg
lb.in	0.1129815	J ou N.m
lb/ft³	0.016020506	g/cm³
lb/in³	27.67783006	g/cm³
lbf	4.448222	N
lbf/in²	1	psi

From	Multiply by	To
long ton	1016.047	kg
M	10^{10}	A
M	3.281	ft
m^2	10.76	ft^2
MBTU	1,000,000	BTU
mm	0.0394	in
mm^2	1.550×10^3	in^2
MMBTU	1,000,000	BTU
MPa	1	N/mm^2
MPa	0.145	ksi
MPa	145	lbf/in^2
MPa.√m	910.06	psi.√in
MPa.√m	0.920	Ksi.√in
N.m	1	J
nm	10^{-9}	m
oz	0.028352707	Kg
Pa	1	N/m^2

From	Multiply by	To
Pa	1.449×10^{-4}	psi
Pa	1.020×10^{-7}	Kg/mm^2
pct (%)	10000	ppm
ppm	0.0001	%
psi	0.001	ksi
psi	0.0068947573	MPa
psi	0.0007030697	kgf/mm^2
psi √in	0.001098829	MPa √m
Rad	57.2958	°
short ton	907.1847	Kg
t	9.8	kN
Th	1	Mcal
Th	4.186×10^6	J
th/h	1.163	kW
W	1	J/s
W	0.001341	HP

- **Useful Data and Constants**

API Pipeline Steels – Mechanical Properties

Grade	Yield Strength, Minimum (MPa)	Yield Strength, Maximum (MPa) only for PSL 2	Ultimate Tensile Strength, Minimum (MPa)	Ultimate Tensile Strength, Maximum (MPa) only for PSL 2
X42	290	496	414	758
X46	317	524	434	758
X52	359	531	455	758
X56	386	544	490	758
X60	414	565	517	758
X65	448	600	531	758
X70	483	621	565	758
X80 (for PSL 2)	552	690	621	827
X90 (for PSL 2)	625	775	695	915
X100 (for PSL 2)	690	840	760	990
X120 (for PSL 2)	830	1050	915	1145

Fuels and Combustion Gases:

- **Density (Gas)**
  . Natural Gas: 0.81 kg/Nm³
  . Butane: 2.44 kg/Nm³
  . Propane: 1.85 kg/Nm³
  . Liquified Petroleum Gas (LPG): 2.29 kg/Nm³
  . Air: 1.27 kg/Nm³

- **Density (Liquid)**
  . Butane: 0.58 kg/l
  . Propane: 0.51 kg/l
  . Liquified Petroleum Gas (LPG): 0.54 kg/Nm³
  . Water: 1.00 kg/Nm³

- **Heat Capacity in Function of Temperature**
  . Heat Capacity [kcal/°C m³] = a + bT [°C]. Values of **a** and **b** for some gases are seen below:

Gas	a	b
C_2H_6	0.600	0.000540
C_3H_8	0.871	0.001226
CH_4	0.380	0.000210
CO	0.302	0.000022
CO_2	0.406	0.000090
H_2	0.301	0.000200
N_2	0.302	0.000022
O_2	0.320	0.000059

- **Net Heating Value**
  . Acetylene (C_2H_2): 13412 Kcal/Nm³
  . Basic Oxygen Steelmaking Off-Gas (OG Gas): 770 kcal/Nm³
  . Blast Furnace Gas: 770 Kcal/Nm³
  . Benzene (C_6H_6): 33,823 Kcal/Nm³
  . Butane (C_4H_{10}): 29,560 Kcal/Nm³
  . Butene/Buthylene (C_4H_8): 27,900 Kcal/Nm³
  . Charcoal: 6,800 kcal/kg
  . Carbon Monoxide (CO): 3,019 Kcal/Nm³
  . Coke Oven Gas: 4,400 Kcal/Nm³
  . Diesel Oil: 10.200 kcal/kg
  . Electricity: 860 kcal/kW
  . Ethane (C_2H_6): 15,236 Kcal/Nm³
  . Ethene/Ethylene (C_2H_4): 14,116 Kcal/Nm³
  . Fuel Oil: 8,640~9,000 kcal/l or 9,600 ~ 10,000 kcal/kg
  . Hexane (C_6H_{14}): 41,132 Kcal/Nm³
  . Hydrogen (H): 2,582 Kcal/Nm³

. Hydrogen Sulfide (H_2S): 5,527 Kcal/Nm³
. i-Butane (C_4H_{10}): 28,317 Kcal/Nm³
. i-Pentane (C_5H_{12}): 34,794 Kcal/Nm³
. Liquified Petroleum Gas (LPG): 25,300 ~ 27,300 kcal/Nm³
. Methane (CH_4): 8,557 Kcal/Nm³
. Natural Gas: 9,000 ~ 9,400 Kcal/Nm³
. Pentane (C_5H_{12}): 34,943 Kcal/Nm³
. Propane (C_3H_8): 21,809 Kcal/Nm³
. Propene/Propylene (C_3H_6): 20,550 Kcal/Nm³
. Toluene (C_7H_8): 40,182 Kcal/Nm³
. Xylene (C_8H_{10}): 46,733 Kcal/Nm³
. Wood: 2,500 kcal/kg

- **Typical Chemical Compositions**

[% vol]	**N_2**	**H_2**	**CH_4**	**C_2H_6**	**C_3H_8**	**CO**	**CO_2**	**O_2**
Coke Oven Gas	3.09	61.55	24.54	0.42	0.06	8.04	0.00	0.26
Natural Gas	1.83	----	87.91	7.08	1.91	----	0.59	0.00

Mathematical Constants

- e: 2.718281828
- Pi: 3.141592654

Physical Constants

- Acceleration of gravity: g = 9.805 m/s²
- Avogrado: N_A = 6.022 x 10^{23} 1/mol
- Boltzmann: k = 1.38065 x 10^{-23} J/K

- Ideal Gas Constant R:
  . 1.98717 cal/(K mol)
  . 82.056 cm^3 atm/(K.mol)
  . 0.082056 l atm/(K.mol)
  . 8.31433 x 10^7 erg/(K.mol)
  . 8.31433 J/(K.mol)
- Stefan-Boltzmann: σ = 5.6704 x 10^{-8} W/(m^2 K^4) or J/(s m^2 K^4)

Physical Properties of Scale (Iron Oxide)

- Thermal Conductivity:
  . Industrial Scale: 3.0 W/(m.K)
  . Hematite (Fe_2O_3): 1.2 W/(m.K)
  . Magnetite (Fe_3O_4): 1.5 W/(m.K)
  . Wustite (FeO): 3.2 W/(m.K)
- Density:
  . Industrial Scale:
    - 4.86 g/cm^3 (Combustol)
    - 5.00 g/cm^3 (Picqué)
    - 5.70 g/cm^3 (Krzyzanowski)
  . Hematite (Fe_2O_3): 4.90 g/cm^3
  . Magnetite (Fe_3O_4): 5.00 g/cm^3
  . Wustite (FeO): 5.70 g/cm^3
  . Bulk, Porous Scale as Raw Material, Room Temperature:
    - 2.40~2.89 t/m^3
    - Stowage Factor: 0,38 m^3/t
- Hardness:
  . Hematite (Fe_2O_3): 1000 HV
  . Magnetite (Fe_3O_4): 320 ~ 500 HV
  . Wustite (FeO): 270 ~ 350 HV
- Iron Content in Scale: 74.6% (Stoichiometric)

- Linear Coefficient of Thermal Contraction:
  . Fe: 19 x 10^{-6} m/°C
  . Wustite: 14 x 10^{-6} m/°C
- Melting Point:
  . Fayalite ($2FeO.SiO_2$): 1177°C
- Specific Heat:
  . Industrial Scale: 766 J/(kg.K) (600°C)
  . Hematite (Fe_2O_3): 980 J/(kg.K)
  . Magnetite (Fe_3O_4): 870 J/(kg.K)
  . Wustite (FeO): 725 J/(kg.K)
- Thermal Expansion Coefficient:
  . Ferrite (0 ~ 900°C): 15.3 x 10^{-6} K^{-1}
  . Hematite (Fe_2O_3):
    - 20 ~ 900°C: 14.9 x 10^{-6} K^{-1}
    - 100 ~ 300°C: 10.8 x 10^{-6} K^{-1}
    - 100 ~ 1000°C: 12.2 x 10^{-6} K^{-1}
    - 400 ~ 800°C: 13.0 x 10^{-6} K^{-1}
  . Magnetite (Fe_3O_4): 1.5 W/(m.K)
    - 25°C: 11.0 x 10^{-6} K^{-1}
    - 400°C: 14.0 x 10^{-6} K^{-1}
    - 550°C: 27.0 x 10^{-6} K^{-1}
  . Wustite (FeO):
    - 100 ~ 1000°C: 12.2 x 10^{-6} K^{-1}
    - 400 ~ 800°C: 15.0 x 10^{-6} K^{-1}

Physical Properties of Steel and its Microstructural Constituents

- Densities:
  . Bulk Steel: 7850 kg/m³
  . Ferrite (Fe α):
    - Caballero: 7882 kg/m³

  - Jablonka: 7870 kg/m³ (20°C)
  . Cementite (Fe_3C):
  - Caballero: 7687 kg/m³
  - Jablonka: 7685 kg/m³ (20°C)
  . NbC: 7790 kg/m³
  . VC: 5700 kg/m³
- Electrical Resistivity at 15.6°C: 17 x 10^{-8} Ω.m
- Emissivity of Polished Metal Surface:
  . 0.07 @ 38°C
  . 0.10 @ 260°C
  . 0.14 @ 540°C
- Emissivity of Oxidized Steel Plate at 15.6°C: 0.80
- Heat Transfer Coefficient at Scale/Steel Interface: 30,000 W/m².K
- Lattice Parameters (Ambient Temperature):
  . Ferrite (pure Fe): 2.866 Å
  . Cementite: a = 4.5246 Å, b = 5.0885 Å, c = 6.7423 Å
- Linear Coefficient of Thermal Expansion:
  . Bulk: 11.7 x 10^{-6} °C^{-1}
  . Ferrite: 1.244 x 10^{-5} °C^{-1}
  . Austenite: 2.065 x 10^{-5} l°C^{-1}
- Melting Point: 1300 ~ 1450°C
- Modulus:
  . Bulk: 159,000 MPa
  . Shear: 83,000 MPa
  . Young: 207,000 MPa
- Poisson's Ratio:
  . Elastic Range: 0.3
  . Plastic Range: 0.5
- Specific Heat: 0.12 cal/g.°C
- Speed of Sound through Steel: 5,490 m/s
- Thermal Conductivity at 15.6°C: 58.9 W/m.K
- Volumetric Coefficient of Thermal Expansion: 35.1 x 10^{-6} °C^{-1}

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