FOREWORD

This is a compilation of some useful mathematical formulas, graphics and data in the area of forming, heat treatment and physical metallurgy of steels. The very first version arose in the early eighties, as a handwritten sheet with a few formulas. Afterwards it was converted to a digital format and eventually posted on-line, hoping that it could be also helpful worldwide. It must be noted that these formulas were compiled at random, generally in a need-to-know basis. So, this Handbook is in permanent construction and very far to be complete. Finally, the author thanks Seok-Jae Lee, Assistant Professor of the Chonbuk National University, Republic of Korea, for his contribution.

DISCLAIMER

The formulas and information compiled in this text are provided without warranty of any kind.

Use them at your own risk!

However, any help regarding the correction of eventual mistakes is appreciated.

A Work in Progress
First On-Line Release: 09 October 2003
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- **Austenite Formation Temperatures**

  . Andrews

  \[
  Ae_1 = 723 - 16.9 \, Ni + 29.1 \, Si + 6.38 \, W - 10.7 \, Mn + 16.9 \, Cr + 290 \, As
  \]
  \[
  Ae_3 = 910 - 203 \sqrt{C} + 44.7 \, Si - 15.2 \, Ni + 31.5 \, Mo + 104 \, V + 13.1 \, W - 30.0 \, Mn + 11.0 \, Cr + 20.0 \, Cu - 700 \, P - 400 \, Al - 120 \, As - 400 \, Ti
  \]

  Notation:
  - \( Ae_1 \): Lower Equilibrium Temperature Between Ferrite and Austenite [°C]
  - \( Ae_3 \): Upper Equilibrium Temperature Between Ferrite and Austenite [°C]
  - **Alloy Content**: [weight %]

  Observations:
  - Both formulas are valid for low alloy steels with less than 0.6% C.


  . Brandis

  \[
  Ac_1 = 739 - 22 \, C - 7 \, Mn + 2 \, Si + 14 \, Cr + 13 \, Mo - 13 \, Ni + 20 \, V
  \]
  \[
  Ac_3 = 902 - 255 \, C - 11 \, Mn + 19 \, Si - 5 \, Cr + 13 \, Mo - 20 \, Ni + 55 \, V
  \]

  Notation:
  - \( Ae_1 \): Lower Equilibrium Temperature Between Ferrite and Austenite [°C]
  - \( Ae_3 \): Upper Equilibrium Temperature Between Ferrite and Austenite [°C]
Alloy Content: [weight %]

Source: BRANDIS, H. "Rechnerische Bestimmung der Umwandlungstemperaturen von niedriglegierten Stählen." TEW - Technische Berichte, Band 1, Heft 1, 1975, 8-10.

. Eldis

\[
Ae_1 = 712 - 17.8 \text{Mn} - 19.1 \text{Ni} + 20.1 \text{Si} + 11.9 \text{Cr} + 9.8 \text{Mo}
\]

\[
Ae_3 = 871 - 254.4 \sqrt{\text{C}} - 14.2 \text{Ni} + 51.7 \text{Si}
\]

Notation:
- \( Ae_1 \): Lower Equilibrium Temperature Between Ferrite and Austenite [°C]
- \( Ae_3 \): Upper Equilibrium Temperature Between Ferrite and Austenite [°C]
- Alloy Content: [weight %]

Observations:
- Both formulas were proposed by ELDIS for low alloy steels with less than 0.6%C.


. Grange

\[
Ae_1 = 1333 - 25 \text{Mn} + 40 \text{Si} + 42 \text{Cr} - 26 \text{Ni}
\]

\[
Ae_3 = 1570 - 323 \text{C} - 25 \text{Mn} + 80 \text{Si} - 3 \text{Cr} - 32 \text{Ni}
\]
Notation:
- **Ae**<sub>1</sub>: Lower Equilibrium Temperature Between Ferrite and Austenite [°F]
- **Ae**<sub>3</sub>: Upper Equilibrium Temperature Between Ferrite and Austenite [°F]
- **Alloy Content**: [weight %]


### . Hougardy

\[
Ac_1 = 739 - 22 \cdot C - 7 \cdot Mn + 2 \cdot Si + 14 \cdot Cr + 13 \cdot Mo - 13 \cdot Ni
\]

\[
Ac_3 = 902 - 255 \cdot C - 11 \cdot Mn + 19 \cdot Si - 5 \cdot Cr + 13 \cdot Mo - 20 \cdot Ni + 55 \cdot V
\]

Notation:
- **Ac**<sub>1</sub>: Lower Temperature of the Ferrite-Austenite Field During Heating [°C]
- **Ac**<sub>3</sub>: Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
- **Alloy Content**: [weight %]


### . Kasatkin

\[
Ac_1 = 723 - 7.08 \cdot Mn + 37.7 \cdot Si + 18.1 \cdot Cr + 44.2 \cdot Mo + 8.95 \cdot Ni + 50.1 \cdot V + 21.7 \cdot Al + 3.18 \cdot W + 297 \cdot S - 830 \cdot N - 11.5 \cdot C \cdot Si - 14.0 \cdot Mn \cdot Si - 3.10 \cdot Si \cdot Cr - 57.9 \cdot C \cdot Mo - 15.5 \cdot Mn \cdot Mo - 5.28 \cdot C \cdot Ni - 6.0 \cdot Mn \cdot Ni + 6.77 \cdot Si \cdot Ni - 0.80 \cdot Cr \cdot Ni - 27.4 \cdot C \cdot V + 30.8 \cdot Mo \cdot V - 0.84 \cdot Cr^2 - 3.46 \cdot Mo^2 - 0.46 \cdot Ni^2 - 28 \cdot V^2
\]

Observations:
- Multiple Correlation Coefficient $r = 0.96$
- Residual Mean-Square Deviation $\sqrt{d_o} = 10.8^\circ C$

$$Ac_3 = 912 - 370 C - 27.4 Mn + 27.3 Si - 6.35 Cr - 32.7 Ni + 95.2 V + 190 Ti + 72.0 Al + 64.5 Nb + 5.57 W + 332 S + 276 P + 485 N - 900 B + 16.2 C Mn + 32.3 C Si + 15.4 C Cr + 48.0 C Ni + 4.32 Si Cr - 17.3 Si Mo - 18.6 Si Ni + 4.80 Mn Ni + 40.5 Mo V + 174 C^2 + 2.46 Mn^2 - 6.86 Si^2 + 0.322 Cr^2 + 9.90 Mo^2 + 1.24 Ni^2 - 60.2 V^2$$

Observations:
- Multiple Correlation Coefficient $r = 0.98$
- Residual Mean-Square Deviation $\sqrt{d_o} = 14.5^\circ C$

$$\Delta T = 188 - 370 C - 7.93 Mn - 26.8 Cr - 33.0 Mo - 23.5 Ni + 52.5 V + 194 Ti + 47.8 Al + 87.4 Nb + 3.82 W + 266 P + 53.0 C Si + 20.7 C Cr + 6.26 Si Cr + 64.2 C Mo + 55.2 C Ni + 10.8 Mn Ni + 1.33 Cr^2 + 8.83 Mo^2 + 1.91 Ni^2 - 37.8 V^2$$

Notation:
- $Ac_1$: Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
- $Ac_3$: Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
- $\Delta T$: Intercritical Temperature Range [°C]
- **Alloy Content**: [weight %]

Observations:
- Multiple Correlation Coefficient $r = 0.97$
- Residual Mean-Square Deviation $\sqrt{d_o} = 16.8^\circ C$
- These equations ($Ac_1$, $Ac_3$, $\Delta T$) are valid within these composition limits: $C \leq 0.83\%$, $Mn \leq 2.0\%$, $Si \leq 1.0\%$, $Cr \leq 2.0\%$, $Mo \leq 1.0\%$, $Ni \leq 3.0\%$, $V \leq 0.5\%$, $W \leq 1.0\%$, $Ti \leq 0.15\%$, $Al \leq 0.2\%$, $Cu \leq 1.0\%$, $Nb \leq 0.20\%$, $P \leq 0.040\%$, $S \leq 0.040\%$, $N \leq 0.025\%$, $B \leq 0.010\%$.

. Kunitake & Katou

\[ Ac_1 = 754.83 - 32.25 \, C - 17.76 \, Mn + 23.32 \, Si + 17.3 \, Cr + 4.51 \, Mo + 15.62 \, V \]

\[ Ac_3 = 920.21 - 394.75 \, C - 14.40 \, Mn + 54.99 \, Si + 5.77 \, Cr + 24.49 \, Mo + 83.37 \, V \]

\[ \delta = 0.0617 + 0.233 \, C + 0.025 \, Mn - 0.049 \, Si + 0.027 \, Cr - 0.050 \, Mo - 0.061 \, V \]

Notation:
- \( Ac_1 \): Lower Temperature of the Ferrite-Austenite Field During Heating [°C]
- \( Ac_3 \): Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
- \( \delta \): Length Change Due to Transformation [%]
- **Alloy Content**: [weight %]

Observations:
- Equation valid within the following alloy range: \( 0.25\% \leq \, C \leq 0.45\%; \, 0.7 \leq \, Mn \leq 1.2\%; \, 0.6 \leq \, Si \leq 3.1\%; \, 0.8 \leq \, Cr \leq 2.9\%; \, 0.2 \leq \, Mo \leq 0.9\%; \, V \leq 0.4\%.


. Lee

\[ A_{cm} = 224.4 + 992.4 \, C - 465.1 \, C^2 + 46.7 \, Cr + 19.0 \, C \, Cr - 6.1 \, Cr^2 + 7.6 \, Mn + 10.0 \, Mo - 6.8 \, Cr \, Mo - 6.9 \, Ni + 3.7 \, C \, Ni - 2.7 \, Cr \, Ni + 0.8 \, Ni^2 + 16.7 \, Si \]

Notation:
- \( A_{cm} \): Upper Equilibrium Temperature Between Ferrite and Cementite [°C]
Alloy Content: [weight %]

Observations:
- Equation valid within the following alloy range: \(0.2\% \leq C \leq 0.7\%; \text{Mn} \leq 1.5\%; \text{Si} \leq 0.3\%; \text{Ni} \leq 2.8\%; \text{Cr} \leq 1.5\%; \text{Mo} \leq 0.6\%\).
- Regression coefficient \(r^2 = 0.998837\); precision interval: \(\pm 3^\circ\text{C}\).


. Park

\[ A_{c3} = 955 - 350C - 25Mn + 51Si + 106Nb + 100Ti + 68Al - 11Cr - 33Ni - 16Cu + 67Mo \]

Notation:
- \(A_{c3}\): Upper Temperature of the Ferrite-Austenite Field During Heating \([^\circ\text{C}]\)
- Alloy Content: [weight %]

Observations:
- Formula specifically developed for TRIP steels.


. Roberts

\[ A_{e3} = 910 - 25Mn - 11Cr - 20Cu + 60Si + 700P - 250Al - F_n \]

Notation:
\( \textbf{Ae}_3 \): Upper Equilibrium Temperature Between Ferrite and Austenite \([\circ C]\)

**Alloy Content**: [weight %]

\( F_n \): value defined according to the table below:

<table>
<thead>
<tr>
<th>C</th>
<th>( F_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>24</td>
</tr>
<tr>
<td>0.10</td>
<td>48</td>
</tr>
<tr>
<td>0.15</td>
<td>64</td>
</tr>
<tr>
<td>0.20</td>
<td>80</td>
</tr>
<tr>
<td>0.25</td>
<td>93</td>
</tr>
<tr>
<td>0.30</td>
<td>106</td>
</tr>
<tr>
<td>0.35</td>
<td>117</td>
</tr>
<tr>
<td>0.40</td>
<td>128</td>
</tr>
</tbody>
</table>


. Trzaska (I)

\[
\text{Ac}_1 = 739 - 22.8C - 6.8Mn + 18.2Si + 11.7Cr - 15Ni - 6.4Mo - 5V - 28Cu
\]

\[
\text{Ac}_3 = 937.3 - 224.5\sqrt{C} - 17Mn + 34Si - 14Ni + 21.6Mo + 41.8V - 20Cu
\]

Notation:

- \( \text{Ac}_1 \): Lower Temperature of the Ferrite-Austenite Field During Heating \([\circ C]\)
- \( \text{Ac}_3 \): Upper Temperature of the Ferrite-Austenite Field During Heating \([\circ C]\)
- **Alloy Content**: [weight %]

. Trzaska (II)

\[ Ac_1 = 742 - 29C - 14\ Mn + 13\ Si + 16\ Cr - 17\ Ni - 16\ Mo + 45\ V + 36\ Cu \]

\[ Ac_3 = 925 - 219\sqrt{C} - 7\ Mn + 39\ Si - 16\ Ni + 13\ Mo + 97\ V \]

Notation:
- **Ac**: Lower Temperature of the Ferrite-Austenite Field During Heating [°C]
- **Ac**\(_3\): Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
- **Alloy Content**: [weight %]

Observations:
- Equation valid within the following alloy range: 0.06% ≤ C ≤ 0.68%; 0.13 ≤ Mn ≤ 2.04%; 0.12 ≤ Si ≤ 1.75%; Ni ≤ 3.85%; Cr ≤ 2.30%; Mo ≤ 1.05%; V ≤ 0.38%; Cu ≤ 0.38.
- Additional validity limitations: Mn + Cr ≤ 3.6; Mn + Cr + Ni ≤ 5.6; Cr + Ni ≤ 5.3; Mn + Ni ≤ 4.5
- **Ac**\(_1\) statistical parameters: regression coefficient: \(r^2 = 0.61\); standard error: ± 15.55°C.
- **Ac**\(_3\) statistical parameters: regression coefficient: \(r^2 = 0.75\); standard error: ± 17.80°C.

- **Austenite Grain Size After Heating**

  . Lee

  \[ d = 76671 \exp \left( - \frac{89098 + 3581C + 1211Ni + 1443Cr + 4031Mo}{RT} \right) t^{0.211} \]

  Notation:
  - \( d \): Austenite Grain Size [μm]
  - \( R \): Universal Gas Constant, 8.314 J/mol.K
  - \( T \): Austenitizing Temperature [K]
  - \( t \): Austenitizing Time [s]

  Observations:
  - Equation valid under the following alloy range: 0.15% ≤ C ≤ 0.41%; 0.73% ≤ Mn ≤ 0.85; 0.20% ≤ Si ≤ 0.25%; Ni ≤ 1.80%; Cr ≤ 1.45%; Mo ≤ 0.45.

- **Austenite Hardenability: Critical Diameter**

  . Dearden & O’Neill

\[
D_i = 6 \times \exp \left[ 7.1 \times \left( C \frac{Mn}{5.87} + \frac{Mo}{3.13} + \frac{Cr}{6.28} + \frac{Si}{18} + \frac{Ni}{15} \right) \right]
\]

Notation:

- **Di**: Critical Diameter [mm]
- **Alloy Content**: [weight %]

- **Austenite Hardenability: Martensite Formation Critical Cooling Rate**

  **Gang**

  \[ V_{c100} = e^{5.3 - 2.53 C - 0.16 Si - 0.82 Mn - 0.95 Cr - 1.87 Mo - 160 B} \]

  Notation:
  - \( V_{c100} \): Critical Cooling Rate to Obtain 100% Martensite Structure [°C/s]
  - **Alloy Content**: [weight %]


  **Ueno**

  For B Steels (free B ≥ 5 ppm):

  \[ V_{c90} = 10^{2.94 - 0.75 \beta} \]

  \[ \beta = 2.7 C + 0.4 Si + Mn + 0.45 Ni + 0.8 Cr + 2 Mo \]

  For B-free Steels:

  \[ V_{c90} = 10^{3.69 - 0.75 \beta'} \]

  \[ \beta' = 2.7 C + 0.4 Si + Mn + 0.45 Ni + 0.8 Cr + Mo \]

  Notation:
  - \( V_{c90} \): Critical Cooling Rate to Obtain 90% Martensite Structure [°C/s]
**Alloy Content:** [weight %]

- **Austenite No-Recrystallization Temperature**

  . Bai 1996

For interpass times less than or equal 12.5 s:

\[
T_{nr} = [88.1 \log([Nb] + 0.31[Ti] + 0.15[Al]) + 1156] \varepsilon^{-0.12} \varepsilon^{-0.01} t^{-0.1}
\]

For interpass times greater 12.5 s and less than or equal 30.0 s:

\[
T_{nr} = [63.5 \log([Nb][C]) + 885] \varepsilon^{-0.12} \varepsilon^{-0.01} t^{0.04}
\]

For interpass times less than 10 s:

\[
\lambda = 3.52 \log(\varepsilon) + 5.3 - (3.10 \log(\varepsilon) + 4.95) \frac{T}{T_{nr}}
\]

\[
RLT = T_{nr} \frac{3.52 \log(\varepsilon) + 5.30}{3.10 \log(\varepsilon) + 4.95}
\]

\[
RST = T_{nr} \frac{3.52 \log(\varepsilon) + 4.70}{3.10 \log(\varepsilon) + 4.95}
\]

For interpass times equal or greater than 10 s and equal or less than 30 s:

\[
\lambda = 3.52 \log(\varepsilon) + 6.4 - (3.10 \log(\varepsilon) + 5.93) \frac{T}{T_{nr}}
\]
\[ RLT = T_{nr} \frac{3.52 \log(\varepsilon) + 6.40}{3.10 \log(\varepsilon) + 5.93} \]

\[ RST = T_{nr} \frac{3.52 \log(\varepsilon) + 5.80}{3.10 \log(\varepsilon) + 5.93} \]

Notation:

- \( T_{nr} \): No-Recrystallization Temperature [°C].
- **Alloy Content**: [weight %]
- \( \varepsilon \): Pass True Strain.
- \( \dot{\varepsilon} \): Pass Strain Rate [s\(^{-1}\)].
- \( t \): Interpass Time [s].
- \( \lambda \): Residual Strain Ratio Between Consecutive Passes.
- \( T \): Pass Temperature [°C].

Observations:

- \( \varepsilon \) must be higher than 0.15 for calculation of RLT.


\[ RLT = 174 \log \left[ Nb \left( C + \frac{12}{14} N_{ef} \right) \right] + 1444 \]

\[ RST = RLT - 75 \]
\[ N_{\text{eff}} = N - \frac{14}{48} Ti \]

Notation:
- **RLT**: Recrystallization Limit Temperature \([^\circ\text{C}]\): full austenite recrystallization between deformation steps is no longer possible below this temperature.
- **RST**: Recrystallization Stop Temperature \([^\circ\text{C}]\): austenite recrystallization stops completely below this temperature.
- **Alloy Content**: [weight %]

Observations:
- \( N_{\text{eff}} \geq 0 \).


. Boratto

\[ T_{\text{nr}} = 887 + 464 C + (6445 Nb - 644 \sqrt{Nb}) + (732 V - 230 \sqrt{V}) + 890 Ti + 363 Al - 357 Si \]

Notation:
- **T_{\text{nr}}**: Temperature of No-Recrystallization \([^\circ\text{C}]\). Maximum temperature at which austenite recrystallizes completely between two deformation passes.
- **Alloy Content**: [weight %]

Observations:
- Equation valid under the following alloy range: \(0.04 \leq C \leq 0.17\%; 0.41 \leq Mn \leq 1.90\%; 0.15 \leq Si \leq 0.50\%; 0.002 \leq Al \leq 0.650\%; Nb \leq 0.060\%; V \leq 0.120\%; Ti \leq 0.110\%; Cr \leq 0.67\%; Ni \leq 0.45\%.


**Fletcher**

\[ T_{\text{nr}} = 849 - 349C + 676\sqrt{Nb} + 337V \]

Observations:
- \( r^2 = 0.72 \)

Notation:
- \( T_{\text{nr}} \): Temperature of No-Recrystallization [°C]. Maximum temperature at which austenite recrystallizes completely between two deformation passes.
- **Alloy Content**: [weight %].


**Fletcher-Bai**

\[ T_{\text{nr}} = 203 - 310C + 657\sqrt{Nb} - 149\sqrt{V} + 683e^{-0.36\varepsilon} \]

Notation:
- \( T_{\text{nr}} \): Temperature of No-Recrystallization [°C]. Maximum temperature at which austenite recrystallizes completely between two deformation passes.
- **Alloy Content**: [weight %].
- \( \varepsilon \): True Strain.

- **Militzer**

\[ T_{nr} = 893 + 910 \text{Nb} \]

Notation:

- **T\text{nr}**: Temperature of No-Recrystallization [°C]. Maximum temperature at which austenite recrystallizes completely between two deformation passes.

**Alloy Content**: [weight %]

Observations:

- Equation valid for 0.020% \( \leq \text{Nb} \leq 0.090\% \).

- **Austenite Solubility Products**

  **General Formula**

  \[
  \log_{10} k_s = \log_{10} \left( \frac{(a_M)^n (a_X)^n}{a_{M, X}} \right) = \log_{10} \left( \frac{[M]^n [X]^n}{[MX]} \right) = \left( \frac{Q}{2.303 RT} \right) + \frac{C}{2.303} = -\frac{A}{T} + B
  \]

  **Notation:**
  - \( M_m X_n \): Precipitate Considered for Calculation
  - \( A_i \): Activity
  - \( M, X \): Alloy Contents [weight %]
  - \( T \): Temperature [K]
  - \( C \): Constant
  - \( A, B \): Constants of the Solubility Product, given in the table below:

<table>
<thead>
<tr>
<th>Precipitate</th>
<th>A</th>
<th>B</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlN</td>
<td>7060</td>
<td>1.55</td>
<td>Narita</td>
</tr>
<tr>
<td></td>
<td>6770</td>
<td>1.03</td>
<td>Irvine</td>
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<tr>
<td></td>
<td>7750</td>
<td>1.80</td>
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<tr>
<td>BN</td>
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<tr>
<td>Cr\textsubscript{23}C\textsubscript{6}</td>
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<td>5.36</td>
<td>Ashby</td>
</tr>
<tr>
<td>Mo\textsubscript{2}C</td>
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<td>Johansen</td>
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<td></td>
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<td>3.04</td>
<td>Meyer</td>
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<tr>
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<td>Value 2</td>
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</tr>
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<tr>
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<td>4.26</td>
<td>Narita</td>
</tr>
</tbody>
</table>

Observations:
- $a_{AmBn}$ is equal to one if the precipitate is pure.
- $a_{AmBn} \leq 1$ if there is co-precipitation with another element.
- The product $[M]^m[X]^n$ (that is, $k_s$) defines the graphical boundary of solubilization in a graph $[M] \times [X]$. 
Sources:


- Values compiled by Rajindra Clement Ratnapuli and Fúlvio Siciliano from assorted references when not specified above.
. Dong

\[
\log[Nb] \left[ C + \frac{12}{14} N \right] = 3.14 + 0.35[Si] - 0.91[Mn] + \frac{1371[Mn] - 923[Si] - 8049}{T}
\]

Notation:
- **T**: Temperature \([\text{K}]\)
- **Alloy Content**: \([\text{weight \%}]\)


. Irvine

\[
\log[Nb] \left[ C + \frac{12}{14} N \right] = 2.26 - \frac{6770}{T}
\]


\[
\log[V][N] = 3.46 + 0.12 Mn - \frac{8330}{T}
\]

Notation:
- **T**: Temperature \([\text{K}]\)
- **Alloy Content**: \([\text{weight \%}]\)

**Koyama**

\[
\log [Nb] [C] = \frac{-7970}{T} + 3.31 + \left(\frac{1371}{T} - 0.900\right) [Mn] - \left(\frac{75}{T} - 0.0504\right) [Mn]^2
\]

\[
\log [Nb] [C] = \frac{-7970}{T} + 3.31 + \left(\frac{735}{T} - 0.348\right) [Si]
\]

\[
\log [Nb] [C] = \frac{-7970}{T} + 3.31 + \left(\frac{1113}{T} - 0.691\right) [Cr] - \left(\frac{38}{T} - 0.0228\right) [Cr]^2
\]

\[
\log [Nb] [C] = \frac{-7970}{T} + 3.31 + \left(\frac{148}{T} - 0.094\right) [Ni] - \left(\frac{8.5}{T} - 0.0068\right) [Ni]^2
\]

Notation:

- **T**: Temperature [K]
- **Alloy Content**: [weight %]


\[
\log [Nb] [N] = \frac{-8500}{T} + 2.89 + \left(\frac{1085}{T} - 0.68\right) [Mn] - \left(\frac{48}{T} + 0.032\right) [Mn]^2
\]
\[
\log [Nb] [N] = \frac{-8500}{T} + 2.89 + \left(\frac{1900}{T} - 1.103\right) [Si]
\]

\[
\log [Nb] [N] = \frac{-8500}{T} + 2.89 + \left(\frac{1290}{T} - 0.77\right) [Cr] - \left(\frac{51}{T} + 0.034\right) [Cr]^2
\]

\[
\log [Nb] [N] = \frac{-8500}{T} + 2.89 + \left(\frac{694}{T} - 0.44\right) [Ni] - \left(\frac{29}{T} - 0.0178\right) [Ni]^2
\]

**Notation:**

- \( T \): Temperature [K]
- \( \text{Alloy Content} \): [weight %]


\[
\log [Nb] [N]^{0.65} [C]^{0.24} = 4.09 - \frac{10400}{T}
\]

**Notation:**

- \( T \): Temperature [K]
- \( \text{Alloy Content} \): [weight %]

- Siciliano

\[
\log[Nb] \left[ C + \frac{12}{14} N \right] = 2.26 + \frac{838 [Mn]^{0.246} - 1730 [Si]^{0.994} - 6440}{T}
\]

Notation:

- **T**: Temperature [K]
- **Alloy Content**: [weight %]

- **Austenite Solubilization: Solid Solution**

  . **Uranga**

Nb microalloyed steel

\[
C_{sol} = -0.123 + 0.985 \, C + 0.07804 \, N - 0.07329 \, Nb + 0.147 \, e^{\frac{-205.7}{T+273}}
\]

\[
N_{sol} = -0.0054 + 0.00403 \, C + 0.752 \, N - 0.04245 \, Nb + 0.0516 \, e^{\frac{-2610.8}{T+273}}
\]

\[
Nb_{sol} = 0.0176 - 0.0116 \, C - 1.172 \, N_{eff} + 0.08878 \, Nb + 60030 \, e^{\frac{-20887.3}{T+273}}
\]

NbTi microalloyed steel

\[
C_{sol} = -0.348 + 0.884 \, C + 0.374 \, N - 0.0157 \, Nb - 0.959 \, Ti + 0.462 \, e^{\frac{-252}{T+273}}
\]

\[
N_{sol} = -0.00168 + 0.00847 \, C + 0.683 \, N - 0.0185 \, Nb - 0.197 \, Ti + 0.0339 \, e^{\frac{-2846}{T+273}}
\]

\[
Nb_{sol} = 0.0126 - 0.166 \, C - 0.936 \, N + 0.365 \, Nb + 0.0241 \, Ti + 38696 \, e^{\frac{-21250}{T+273}}
\]
\[ Ti_{sol} = 0.001515 - 0.00337 C - 0.304 N - 0.00157 Nb - 0.0914 Ti + 943570 e^{\left(-30377 \right) / \left(T+273\right)} \]

Notation:
- **Alloy Content**: [weight %]
- **T**: Temperature [°C]

Observations:
- Equations fitted using Thermocalc predictions (TCFE6 Database).
- Useless temperature range: 900 – 1300°C.
- Useless composition range: 0.05% ≤ C ≤ 0.10%, 0.020% ≤ Nb ≤ 0.100%, 0.002% ≤ Ti ≤ 0.025% (except Nb microalloyed steels) and 0.0030 ≤ N ≤ 0.0120.

- **Austenite Solubilization: Temperature**

  . **General Formula**

  \[
  T_d (°C) = \frac{A}{B - \log_{10}(a_A)^m(a_B)^n} - 273
  \]

  Notation:
  - \(A_mB_n\): Precipitate considered for calculation
  - \(T_d\): Solubilization temperature [°C]
  - \(a_x\): Alloy content [weight %]
  - \(A, B\): Constants of the solubility product, given in the table at the topic *Austenite Solubilization Products*.

  . **Chastukhin**

  \[
  T_d = \log_{10}([C]^{0.6}[Nb]^{1.6}) + 33.86[Si] - 1.48[Mn] - 2.73[Cr] + 17.1 + 3.61\frac{[Ti]}{[N]} + 1657
  \]

  Notation:
  - \(T_d\): Dissolution temperature of NbCN [°C]
  - **Alloy Content**: [weight %]


  . **Uranga**
\[ T_d = \frac{-20587.3}{\ln\left(\frac{-0.0176 + 0.116[C] + 1.172 N_{eff} + 0.9122 Nb}{60030}\right)} - 273 \]

\[ N_{eff} = N - \frac{12}{48} Ti \]

Notation:
- \( T_d \): Dissolution temperature of NbCN [°C]
- **Alloy Content**: [weight %]

Observations:
- \( N_{eff} \geq 0 \).
- Formula developed for NbTi microalloyed steels.

- **Austenite Transformation Temperatures: Ferrite Start and Finish**

  . Blás

  \[
  Ar_3 = 903 - 328C - 102 Mn + 116 Nb - 0.909v
  \]

  Notation:
  
  - \( Ar_3 \): Ferrite Start Temperature [°C]
  - **Alloy Amount**: [weight %]
  - \( v \): Cooling Rate [°C/s]

  Observations:
  
  - This formula was determined using temperature data got from samples cooled directly from hot rolling experiments. Thus it includes the effects of hot forming over austenite decomposition.
  - Useful range: 0.024-0.068% C, 0.27-0.39% Mn, 0.004-0.054% Al, 0.000-0.094% Nb, 0.0019-0.0072% N, 1.0-35°C/s
  - \( r = 0.934; \) Root Mean Square Deviation = 5°C


  . Choo

  \[
  Ar_3 = 925.95 - 494.74 C - 64.8 Mn - 10 v^{0.5}
  \]

  Notation:
  
  - \( Ar_3 \): Ferrite Start Temperature [°C]
  - **Alloy Amount**: [weight %]
  - \( v \): Cooling Rate [°C/s]

**Choquet**

\[ \text{Ar}_3 = 902 - 527 \ C - 62 \ Mn + 60 \ Si \]

Notation:
- \( \text{Ar}_3 \): Ferrite Start Temperature [°C]
- **Alloy Amount**: [weight %]

Observations:
- This formula was determined using data got from samples cooled directly from hot rolling experiments. Thus it includes the effects of hot forming over austenite decomposition.


**Gang**

\[ \text{Ar}_3 = 901 - 325 \ C - 92 \ Mn - 126 \ Cr - 67 \ Ni - 149 \ Mo \]

Notation:
- \( \text{Ar}_3 \): Ferrite Start Temperature [°C]
- **Alloy Content**: [weight %]

Observations:
- The previous conditioning of the steel samples that supplied data for the deduction of this formula is unknown.

. **Higashi**

\[
Ar_3 = 901 - 325C - 92Mn + 33Si - 46(Ni + Cr + Cu + Mo)
\]

**Notation:**
- \(Ar_3\): Ferrite Start Temperature [°C]
- **Alloy Content**: [weight %]

**Observations:**
- The previous conditioning of the steel samples that supplied data for the deduction of this formula is unknown.


. **Kariya**

\[
Ar_3 = 910 - 203\sqrt{C} - 30Mn + 44.7Si - 11Cr + 31.5Mo - 15.2Ni
\]

**Notation:**
- \(Ar_3\): Ferrite Start Temperature [°C]
- **Alloy Amount**: [weight %]

. Kotsar

\[ Ar_3 = 879.2 - 94.24 \, C - 21.13 \, Si - 25.56 \, Mn + 47.71 \, Cr + 16.44 \, Ni \]

\[ Ar_1 = 729.2 - 9.24 \, C + 12.13 \, Si - 5.56 \, Mn + 17.71 \, Cr - 46.44 \, Ni \]

Notation:
- \( Ar_3 \): Ferrite Start Temperature [°C]
- \( Ar_1 \): Ferrite Finish Temperature [°C]
- **Alloy Content**: [weight %]

Observations:
- \( Ar_3 \) equation: Maximum Error = 85.5°C (11.9%)  
- \( Ar_1 \) equation: Maximum Error = 35.7°C (4.7%)


. Leysen

\[ Ar_3 = 901 - 830 \, C - 61 \, Mn + 71 \, Si + 362 \, P \]

Notation:
- \( Ar_3 \): Ferrite Start Temperature [°C]
- **Alloy Content**: [weight %]

. Lotter

\[ Ar_3 = 834.8 - 251C - 103.2 Mn + 60.5 Si - 69.7 Cr - 105.5 Mo + 204.5 Nb + 202Ti + 9.0 B + 11.86 \sinh^{-1}(\Delta t_{8/5}) \]

Observations:
- This formula was determined using data got from samples submitted to a normalizing rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; final cooling.
- \( r = 0.92 \); Root Mean Square Deviation = 21°C

\[ Ar_3 = 884.2 - 331C - 98.7 Mn + 65.2 Si - 75.9 Cr - 97.4 Cu - 76.7 Mo - 322.6 Nb + 158Ti + 290V + 9.28 \sinh^{-1}(\Delta t_{8/5}) \]

Observations:
- This formula was determined using data got from samples submitted to a thermomechanical rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; 35% @800°C; final cooling.
- \( r = 0.88 \); Root Mean Square Deviation = 23°C

Notation:
- \( Ar_3 \): Ferrite Start Temperature [°C]
- **Alloy Amount**: [weight %]
- \( \Delta t_{8/5} \): Time Between 800°C and 500°C [s]


. Lutsenko
\[ \text{Ar}_3 = 913.7 - 207.13 C - 46.6 Mn + 110.54 Cr + 108.1 N \]

\[ \text{Ar}_1 = 741.7 - 7.13 C - 14.09 Mn + 16.26 Si + 11.54 Cr - 49.69 Ni \]

Notation:
- \( \text{Ar}_3 \): Ferrite Start Temperature [°C]
- \( \text{Ar}_1 \): Ferrite Finish Temperature [°C]
- **Alloy Amount**: [weight %]


**Miettinen**

Carbon Steels:

\[ \text{Ar}_3 = 888.26 - 581.45 C + 76.66 Si - 55.10 Mn + 553.59 C C - 211.01 C Si - 31.13 C Mn + 4.04 GS - 6.37 \sqrt{C R} - 4.88 \ln (CR) \]

Alloy Steels:

\[ \text{Ar}_3 = 892.15 - 523.35 C + 86.50 Si - 65.88 Mn - 46.85 Cr + 21.82 Mo - 45.17 Ni + 405.05 C C - 180.54 C Si - 3.65 C Mn + 92.20 C Cr - 69.66 C Mo + 37.50 C Ni + 2.43 GS - 5.18 \sqrt{C R} - 6.78 \ln (CR) \]

Notation:
- \( \text{Ar}_3 \): Ferrite Start Temperature [°C]
- **Alloy Amount**: [weight %]
GS = Austenite Grain Size [ASTM units]
CR = Cooling Rate [°C/s]

Observations:
- Carbon Steels: r = 0.97, average error = 14°C. Useful range: 0.05 ≤ C ≤ 0.60%, 0.30 ≤ Mn ≤ 2.00, 0.15 ≤ Si ≤ 0.70%, Cr < 0.5%, 0.2% Mo, 0.5% Ni, 1 ≤ ASTM GS ≤ 10 and CR ≥ 0.05°C/s.
- Alloy Steels: r = 0.95, average error = 16°C. Useful range: 0.05 ≤ C ≤ 0.60%, 0.30 ≤ Mn ≤ 2.00, 0.15 ≤ Si ≤ 0.70%, Cr ≤ 2.0%, Mo ≤ 0.5%, Ni ≤ 3.5%, 1 ≤ ASTM GS ≤ 10 and CR ≥ 0.05°C/s.


Mintz

First Proposal:

\[ \text{Ar}_i = 833.6 - 190.6 \, C - 67.4 \, Mn + 1522 \, S - 2296 \, N_{ti} - 1532 \, Nb + 7.91 \, d^{-1/2} - 0.117 \, CR \]

\[ N_{ti} = N_t - \frac{T_i}{3.5} \]

Observations:
- Useful range: 0.04-0.75% C, 0.30-1.60% Mn, 0.02-0.49% Si, 0.014-0.085% Al, 0.00-0.31% Nb, 0.004-0.008% N, 0.003-0.032% S, d: 0.070-0.950 mm, CR: 25-200°C/min
- r = 0.949; Root Mean Square Deviation = 15.9°C. The coefficients for N_{ti} (82.2%), d (87.4%) and CR (92.8%) were not significant for a 95% minimum confidence level.
- The unexpected positive effect of S can be due to enhanced nucleation of ferrite at sulphides.

Second Proposal:
\[ Ar_s = 868 - 181C - 75.8Mn + 1086S - 3799N_t - 1767Nb - 0.0933CR \]

\[ Ar_s = 862 - 182C - 76.1Mn + 1121S - 1804Nb + 1168Ti - 2852N_t - 0.0084CR \]

\[ N_{Ni} = N_t - \frac{Ti}{3.5} \]

Notation:
- **Ar₃**: Ferrite Start Temperature [°C]
- **Alloy Amount**: [weight %]
- **Nₜ**: Total Nitrogen Content [weight %]
- **d**: Austenite Grain Diameter [mm]
- **Nₜ**: Total Nitrogen Content [weight %]
- **CR**: Cooling Rate [°C/min]

Observations:
- This formula was determined using temperature data got from non-hot deformed samples and includes TRIP steels.
- Useful range: 0.04-0.75% C, 0.31-2.52% Mn, 0.01-1.22% Si, 0.00-1.55% Al, 0.000-0.042% Nb, 0.0012-0.014% N, 0.002-0.110% P, 0.001-0.032% S, d: 0.1-1.0 mm, CR: 10-200°C/min
- \( r = 0.939 \); Root Mean Square Deviation = 18.1°C.

Third Proposal: Specific for TRIP Steels

\[ Ar_s = 870 - 586C - 630P + 18.1Al - 8151Nb \]

Observations:
- Useful range: 0.12-0.22% C, 1.02-2.52% Mn, 0.006-1.05% Si, 0.022-1.55% Al, 0.000-0.026% Nb, 0.0012-0.014% N, 0.010-0.110% P, 0.001-0.0056% S, d: 0.1-1.0 mm, CR: 30 and (mostly) 60°C/min
- $r = 0.957$; Root Mean Square Deviation = 16.6°C.

Notation:

- **Ar₃**: Ferrite Start Temperature [°C]
- **Alloy Amount**: [weight %]
- **N_I**: Total Nitrogen Content [weight %]
- **d**: Austenite Grain Diameter [mm]
- **CR**: Cooling Rate [°C/min]


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**Okamoto**

$$Ar₃ = 901 - 325 C - 92 Mn + 33 Si + 287 P + 40 Al - 20 Cr$$

Notation:

- **Ar₃**: Ferrite Start Temperature [°C]
- **Alloy Amount**: [weight %]


---

**Ouchi**

$$Ar₃ = 910 - 310 C - 80 Mn - 20 Cu - 15 Cr - 55 Ni - 80 Mo + 0.35 (h - 8)$$

Notation:
\( \textbf{Ar}_3 \): Ferrite Start Temperature [°C]

\textbf{Alloy Content}: [weight %]

\( h \): Plate Thickness [mm]

Observations:
- This formula was determined using temperature data got from samples of Nb microalloyed steels cooled directly from hot rolling experiments. Thus it includes the effects of hot forming over austenite decomposition.


. Pickering

\[
\text{Ar}_3 = 910 - 230 \, C - 21 \, Mn - 15 \, Ni + 32 \, Mo + 45 \, Si + 13 \, W + 104 \, V
\]

Notation:
- \( \text{Ar}_3 \): Ferrite Start Temperature [°C]
- \textbf{Alloy Content}: [weight %]

Observations:
- Applicable to Plain C Steels.


. Proprietary #1

\[
\text{Ar}_3 = 879.4 - 516.1 \, C - 65.7 \, Mn + 38.0 \, Si + 274.7 \, P
\]
Notation:

$\text{Ar}_3$: Ferrite Start Temperature [°C]

\textbf{Alloy Content}: [weight %]

Source: Unknown.

. \textbf{Proprietary #2}

\[ \text{Ar}_1 = 706.4 - 350.4 \cdot C - 118.2 \cdot Mn \]

Notation:

$\text{Ar}_1$: Ferrite Finish Temperature [°C]

\textbf{Alloy Content}: [weight %]

Observations:

- The previous conditioning of the steel samples that supplied data for the deduction of this formula is unknown.
- Samples cooled at 20°C/s.

Source: Unknown.

. \textbf{Salganik}

\[ Ar_3 = 735.6 + 180.1 \cdot (C + Cr) + 1206.9 \cdot (S + P) - 10.9 \cdot (Si + Mn + Ni + Cu + Mo) + 755.3 \cdot (Al + N) - 328.8 \cdot (V + Nb + Ti) \]

\[ Ar_1 = 576.8 + 195.7 \cdot (C + Cr) + 3022.6 \cdot (S + P) - 17.5 \cdot (Si + Mn + Ni + Cu + Mo) + 1040 \cdot (Al + N) - 440.6 \cdot (V + Nb + Ti) \]

Notation:
**Ar₃**: Ferrite Start Temperature [°C]

**Ar₁**: Ferrite Finish Temperature [°C]

**Alloy Content**: [weight %]

**Observations:**
- Equations fitted with data from 10 steels
- Formulas valid within the following range: 
  0.060% ≤ C ≤ 0.130%, 0.59% ≤ Mn ≤ 1.65%, 0.24% ≤ Si ≤ 0.46%, 
  0.007% ≤ P ≤ 0.012%, 0.001% ≤ S ≤ 0.025%, 0.020% ≤ Cr ≤ 0.070%, 0.02% ≤ Ni ≤ 0.23%, 0.03% ≤ Cu ≤ 0.22%, 
  0.008% ≤ Al ≤ 0.059%, Nb ≤ 0.048%, Ti ≤ 0.023%, V ≤ 0.048%, Mo ≤ 0.19%, 0.0020% ≤ N ≤ 0.007%
- **Ar₃** equation: \( r^2 = 0.955, \) Standard Error of Estimate = 4.64%
- **Ar₁** equation: \( r^2 = 0.976, \) Standard Error of Estimate = 4.01%
- Applicable to low C microalloyed steels for large diameter pipes.


**. Santos**

\[
Ar_3 = 874.44 - 512.0465C - 40.915Mn + 23.075Si + 567.126C^2 - 199.551C\cdot Mn + 265.797C\cdot Si + 4.148A - 1.03\sqrt{CR} - 11.334\ln(CR)
\]

\[
A = -2\ln\left(\frac{0.002d_f}{\ln(2)}\right)
\]

**Notation:**
- **Ar₃**: Ferrite Start Temperature [°C]
- **Alloy Content**: [weight %]
- **A**: Austenite Grain Size [ASTM units]
- **CR**: Continuous Cooling Rate [°C/s]
- **d_f**: Austenite Grain Size [μm]
Observations:
- Equation fitted with data from 94 points; $r^2 = 0.9888$
- Applicable to plain C Steels.


**Schacht**

\[
Ar_3' = 811 - 255C - 7Mn + 19Si
\]

\[
Ar_3 = Ar_3' - 19 \nu_r^{0.481} - 0.5 e^{\frac{0.042 d_{\gamma} + 7.8}{2.7402}}
\]

\[
Ar_1 = 739 - 22C - 7Mn + 2Si
\]

Notation:
- $Ar_3'$: Ferrite Start Temperature without Undercooling [°C]
- $Ar_3$: Ferrite Start Temperature with Undercooling [°C]
- **Alloy Content**: [weight %]
- $\nu_r$: Continuous Cooling Rate [°C/s]
- $d_{\gamma}$: Austenite Grain Size [μm]

Observations:
- Equation fitted with data from 94 points; $r^2 = 0.9888$
- Applicable to plain C Steels.

. Sekine

\[ Ar_3 = 868 - 396 \, C - 68.1 \, Mn + 24.6 \, Si - 36.1 \, Ni - 24.8 \, Cr - 20.7 \, Cu \]

Notation:

- **Ar**\(_3\): Ferrite Start Temperature [°C]
- **Alloy Content**: [weight %]

Observations:
- This formula was determined using temperature data got from samples cooled directly from hot rolling experiments. Thus it includes the effects of hot forming over austenite decomposition.
- Precision: ±13°C.


. Shiga

\[ Ar_3 = 910 - 273 \, C - 74 \, Mn - 56 \, Ni - 16 \, Cr - 9 \, Mo - 5 \, Cu \]

Notation:

- **Ar**\(_3\): Ferrite Start Temperature [°C]
- **Alloy Content**: [weight %]

Observations:
- This formula was determined using temperature data got from samples cooled directly from hot rolling experiments. Thus it includes the effects of hot forming over austenite decomposition.


. **Trzaska (I)**

\[
Ar_3 = 857 - 257C - 69Mn + 23Si - 38Ni - 20Cr - 20Mo + 34V + 26Cu + 0.07T_A - 17v_R^{0.25}
\]

Notation:
- \(Ar_3\): Ferrite Start Temperature [°C]
- **Alloy Content**: [weight %]
- \(T_A\): Austenitizing Temperature [°C]
- \(v_R\): Cooling Rate [°C/min]

Observations:
- Formula valid within the following range: \(0.21\% \leq C \leq 0.68\%, 0.28\% \leq Mn \leq 2.00\%, 0.13\% \leq Si \leq 1.90\%, Cr \leq 2.5\%, Ni \leq 3.85\%, Mo \leq 1.05\%, V \leq 0.38\%\) and \(Cu \leq 0.38\%\).
- Error = 19.5°C, \(r = 0.86\).


. **Trzaska (II)**

\[
Ar_3 = 1.375Ac_3 - 2.3CR - 339
\]

\[
Ac_3 = 937.3 - 224.5\sqrt{C} - 17Mn + 34Si - 14Ni + 21.6Mo + 41.8V - 20Cu
\]
Observations:
- Standard Error = 11°C, $r^2 = 0.990$.

$$\text{Ar}_1 = 202.75 \ CE^{-0.25} - 2.3 \ CR + 402$$

$$CE = C + \frac{Mn}{6} + \frac{Cr + Mo + V}{5} + \frac{Cu + Ni}{15}$$

Observations:
- Standard Error = 11°C, $r^2 = 0.986$.

Notation:
- $\text{Ac}_3$: Upper Temperature of the Ferrite-Austenite Field During Heating [°C]
- $\text{Ar}_3$: Ferrite Start Temperature [°C]
- $\text{Ar}_1$: Ferrite Finish Temperature [°C]
- Alloy Content: [weight %]
- $CE$: Equivalent Carbon [weight %]
- $CR$: Cooling Rate [°C/s]

Observations:
- Formulas valid within the following range: $0.029\% \leq C \leq 0.73\%$, $0.27\% \leq Mn \leq 0.66\%$, $0.03\% \leq Si \leq 0.22\%$, $2^\circ C/s \leq CR \leq 8^\circ C/s$


- Vanderschueren
\( \text{Ar}_3 = 865 - 280 \, C - 60 \, Mn + 60 \, Si + 600 \, P \)

Notation:
- \( \text{Ar}_3 \): Ferrite Start Temperature [°C]
- **Alloy Content**: [weight %]


Yuan

Non-Deformed Austenite:

\[
\text{Ar}_3 = 370 \exp \left( -\frac{\sqrt{D_r}}{6.7} \right) - 325 \, CR^{0.1} - 5649 \, Nb + 78194 \, Nb^2 + 1019
\]

Notation:
- \( \text{Ar}_3 \): Ferrite Start Temperature [°C]
- **CR**: Continuous Cooling Rate [°C/s]
- **Nb**: Niobium content [weight %]

Observations:
- This formula was determined using temperature data got from non-hot deformed samples.
- Base steel: 0.11% C, 1.20% Mn, 0.20% Si, 0.005% N. Useful range: 0.000-0.038% Nb, CR: 0.5-30°C/s.

Deformed Austenite:
\[ Ar_3 = 370 \exp \left( -\frac{\sqrt{D_y}}{6.7} \right) - 198CR^{0.1} - 6646\text{Nb} - 2327\text{Nb}^2 + 66\left( \frac{1}{t_{0.05}} + \Delta \varepsilon \right) + 830 \]

Notation:
- \( Ar_3 \): Ferrite Start Temperature [°C]
- \( CR \): Continuous Cooling Rate [°C/s]
- \( Nb \): Niobium content [weight %]
- \( t_{0.05} \): Nb(CN) Precipitation Start Time [s]
- \( \Delta \varepsilon \): Residual Strain in Austenite

Observations:
- This formula was determined using temperature data got from samples cooled directly from hot rolling experiments. Thus it includes the effects of hot forming over austenite decomposition.
- Base steel: 0.11% C, 1.20% Mn, 0.20% Si, 0.005% N. Useful range: 0.000-0.038% Nb, CR: 0.5-30°C/s. See reference for details about the calculation of \( t_{0.05} \) and \( \Delta \varepsilon \), which requires external models.


. Záhumenský

\[ Ar_3 = 914 - 6.85\text{CR} - 650\text{C} - 134\text{Mn} + 179\text{Si} \]
\[ Ar_1 = 814 - 9.08\text{CR} - 532\text{C} - 121\text{Mn} + 165\text{Si} \]

Notation:
- \( Ar_3 \): Ferrite Start Temperature [°C]
- \( Ar_1 \): Ferrite Finish Temperature [°C]
- \( CR \): Continuous Cooling Rate [°C/s]
**Alloy Content**: [weight %]

Observations:
- This formula was determined using data got from CMn, HSLA, IF, DP and TRIP Steels.
- Maximum value of cooling rate: 15°C/s.
- Nb, Ti and Al contents were excluded from the equation due to statistical reasons.
- $Ar_3$: $r^2 = 0.91$
- $Ar_1$: $r^2 = 0.90$


---

*Zhao*

$$M_a = 820 - 603.76 \, C + 247.13 \, C^2 - 66.24 \, Mn - 55.72 \, Ni + 3.97 \, Ni^2 - 0.151 \, Ni^3 - 31.10 \, Cr + 2.348 \, Cr^2 - 24.29 \, Mo - 31.88 \, Cu - 0.196 \, Co +$$
$$+ 0.165 \, Co^2 - 0.00255 \, Co^3 - 28.01 \, Ru$$

Notation:
- $M_a$: Massive Ferrite Start Temperature [°C]
- **Alloy Content**: [weight %]

Austenite Transformation Temperatures: Pearlite Start and Finish

Miettinen

Carbon Steels:

\[ P_s = 776.89 - 399.13 C - 53.65 Si - 49.80 Mn + 501.46 C C + 39.63 C Si - 3.09 C Mn + 3.35 GS - 7.05 \sqrt{CR} - 3.22 \ln (CR) \]

Alloy Steels:

\[ P_s = 770.75 - 352.88 C - 18.19 Si - 72.46 Mn - 6.46 Cr - 16.03 Mo - 40.53 Ni + 391.71 C C - 9.37 C Si + 40.02 C Mn + 23.42 C Cr - 90.83 C Mo + 21.84 C Ni + 3.49 GS - 3.63 \sqrt{CR} - 7.71 \ln (CR) \]

Observations:
- Carbon Steels: \( r = 0.93 \), average error = 15\(^\circ\)C.
- Alloy Steels: \( r = 0.89 \), average error = 18\(^\circ\)C.

Carbon Steels:

\[ P_f = 738.88 - 375.73 C + 59.04 Si - 85.65 Mn + 468.34 C C - 185.53 C S + 68.02 C Mn + 3.35 GS - 12.41 \sqrt{CR} - 5.28 \ln (CR) \]

Alloy Steels:

\[ P_s = 729.77 - 353.70 C + 97.38 Si - 104.28 Mn - 11.54 Cr - 39.85 Mo - 44.17 Ni + 405.21 C C - 237.62 C Si + 102.20 C Mn + 32.89 C Cr - 96.79 C Mo + 27.23 C Ni + 3.49 GS - 7.52 \sqrt{CR} - 10.61 \ln (CR) \]
Observations:
- Carbon Steels: \( r = 0.85 \), average error = 10°C.
- Alloy Steels: \( r = 0.78 \), average error = 10°C.

Notation:
- \( Ar_3 \): Ferrite Start Temperature [°C]
- **Alloy Amount**: [weight %]
- **GS** = Austenite Grain Size [ASTM units]
- **CR** = Cooling Rate [°C/s]

Observations:
- Carbon Steels. Useful range: \( 0.05 \leq C \leq 0.60\% \), \( 0.30 \leq Mn \leq 2.00 \), \( 0.15 \leq Si \leq 0.70\% \), \( Cr < 0.5\% \), \( 0.2\% \) Mo, \( 0.5\% \) Ni, \( 1 \leq \) ASTM GS \( \leq 10 \) and \( CR \geq 0.05°C/s \).
- Alloy Steels. Useful range: \( 0.05 \leq C \leq 0.60\% \), \( 0.30 \leq Mn \leq 2.00 \), \( 0.15 \leq Si \leq 0.70\% \), \( Cr < 2.0\% \), \( Mo \leq 0.5\% \), \( Ni \leq 3.5\% \), \( 1 \leq \) ASTM GS \( \leq 10 \) and \( CR \geq 0.05°C/s \).


**Trzaska**

\[
P_i = 780 - 30C - 65Mn + 24Si - 29Ni - 26Mo - 21Cu - 17v_{CR}^{0.25}
\]

Notation:
- \( Ar_3 \): Ferrite Start Temperature [°C]
- **Alloy Content**: [weight %]
- **v_{CR}**: Cooling Rate [°C/min]

Observations:
- Formula valid within the following range: $0.21\% \leq C \leq 0.68\%$, $0.28\% \leq \text{Mn} \leq 2.00\%$, $0.13\% \leq \text{Si} \leq 1.90\%$, $\text{Cr} \leq 2.5\%$, $\text{Ni} \leq 3.85\%$, $\text{Mo} \leq 1.05\%$, $\text{V} \leq 0.38\%$ and $\text{Cu} \leq 0.38\%$.
- Error = 19.4°C, $r = 0.80$.

- Austenite Transformation Temperatures: Bainite Start and Finish

  . Bodnar #1

\[ B_s = 844 - 597 \, C - 63 \, Mn - 16 \, Ni - 78 \, Cr \]

Notation:
- \( B_s \): Bainite Start Temperature [°C]
- **Alloy Amount**: [weight %]


  . Bodnar #2

\[ B_s = 719 - 127 \, C - 50 \, Mn - 31 \, Ni - 27 \, Cr - 61 \, Mo \]

Notation:
- \( B_s \): Bainite Start Temperature [°C]
- **Alloy Amount**: [weight %]

Observations:
- Equation developed using data got from continuous cooled samples.


  . Kang
\[ B_s = 634.8 - 193.1 \, C + 102.4 \, C^2 - 31.2 \, Mn - 4.6 \, Si - 18.6 \, Ni - 32.4 \, Cr - 15.6 \, Mo + 10.36 \ln(d_g) \]

Notation:
- \( B_s \): Bainite Start Temperature [°C]
- \( d_g \): Austenite Grain Size [microns]
- **Alloy Amount**: [weight %]

Observations:
- Reliable chemical composition range: 0.10-1.00% C, 0.17-1.91% Mn, 0.40% Si max, 2.10% Ni max, 2.16% Cr max, 1.96% Mo max.
- Reliable prior austenite grain size range: 1.62-6.70 microns


---

\[ B_s = 656 - 57.7 \, C - 35 \, Mn - 75 \, Si - 15.3 \, Ni - 34 \, Cr - 41.2 \, Mo \]

Notation:
- \( B_s \): Bainite Start Temperature [°C]
- **Alloy Amount**: [weight %]

Observations:
- This is a modification of Steven & Haynes’ formula using isothermal transformation diagrams determined for low and high alloy steels produced by U.S. Steel.

. Kunitake

\[ B_s = 732 - 202 \, C - 85 \, Mn + 216 \, Si - 37 \, Ni - 47 \, Cr - 39 \, Mo \]

Notation:
- \( B_s \): Bainite Start Temperature [°C]
- **Alloy Amount**: [weight %]

Observations:
- This author concluded that the measured \( B_s \) temperature for steels with a greater Ni or Cr content is much higher than that predicted by Steven & Haynes.
- Reliable chemical composition range: 0.11~0.56% C, 0.34~1.49% Mn, 0.14~0.40% Si, 0.07~1.99% Mo, 0.14~4.80% Cr, 0.23~4.33% Ni.
- Error = 10.5°C; correlation coefficient \( r^2 = 0.97 \).


. Leach

\[ WB_s = 850 - 206 \, C - 78 \, Mn - 33 \, Ni - 70 \, Cr - 75 \, Mo - 61 \, Si \]

Notation:
- **WB_s**: Widmanstätten Ferrite/Bainite Start Temperature [°C]
- **Alloy Amount**: [weight %]

Observations:
- RMS Error = 70-76°C.

. Lee #1

\[ B_s = 984.4 - 361.9 \, C + 261.9 \, C^2 - 28.3 \, Mn + 43.7 \, Si \]

Notation:
- \( B_s \): Bainite Start Temperature [K]
- **Alloy Amount**: [weight %]

Observations:
- Formula specifically developed for TRIP steels.


. Lee #2

\[ B_s = 745 - 110 \, C - 59 \, Mn - 39 \, Ni - 68 \, Cr - 106 \, Mn \, Ni - 6 \, Cr^2 + 29 \, Mo^2 \]

Notation:
- \( B_s \): Bainite Start Temperature [°C]
- **Alloy Amount**: [weight %]

Observations:
- This formula is based in the equations of Steven & Haynes, Kirkaldy and Kunitake & Okada, as well data from many time-temperature diagrams for several steels, including low alloy steels and steels with Ni and Cr

- Reliable chemical composition range: 0.10-0.80% C, 0.26-1.63% Mn, 0.13-0.67% Si, 0.00-1.96% Mo, 0.00-4.48% Cr, 0.00-4.34% Ni.


. Li

\[
B_s = 637 - 58C - 35Mn - 15Ni - 34Cr - 41Mo
\]

Notation:
- **B_s**: Bainite Start Temperature [°C]
- **Alloy Amount**: [weight %]

Observations:
- This formula is a modification of Kirkaldy’s B_s equation.
- It assumes that Si amount is constant and equal to 0.25%, as most low alloy steels exhibit a content of this alloy element in this order of magnitude.
- Reliable chemical composition range: 0.20-0.41% C, 0.31-1.01% Mn, 0.10-0.28% Si, 0.00-0.44% Mo, 0.02-0.98% Cr, 0.02-3.04% Ni, 0.05-0.11% Cu.


. Lotter

\[
B_s = 809.8 - 558C - 96.4Mn - 38.7Si - 111.1Cr - 76.3Mo - 723.2Nb - 530Ti + 6.12\sinh^{-1}\left(\Delta T_s/5\right)
\]
Observations:
- This formula was determined using data got from samples submitted to a normalizing rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; final cooling.
- \( r = 0.72; \) Root Mean Square Deviation = 29°C

\[
B_s = 705.2 - 180C - 53.8 Mn + 26.9 Si - 92.9 Cr - 72.0 Mo - 248V + 3.96 \sinh\left(\frac{\Delta t}{85}\right)
\]

Observations:
- This formula was determined using data got from samples submitted to a thermomechanical rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; 35% @800°C; final cooling.
- \( r = 0.64; \) Root Mean Square Deviation = 27°C

Notation:
- \( B_s \): Bainite Start Temperature [°C]
- **Alloy Amount**: [weight %]
- \( \Delta t_{8/5} \): Time Between 800°C and 500°C [s]


. Miettinen

Carbon Steels:

\[
B_s = 537.86 - 132.17 C + 138.04 Si + 34.65 Mn + 251.53 C C - 831.99 C Si - 7.46 C Mn
\]
Alloy Steels:

\[
B_s = 578.19 - 100.75 \, C + 81.32 \, Si + 7.12 \, Mn + 11.85 \, Cr + 97.71 \, Mo - 14.44 \, Ni + 228.79 \, C \, C - 449.69 \, C \, Si \\
- 57.11 \, C \, Mn - 57.17 \, C \, Cr - 166.88 \, C \, Mo - 18.75 \, C \, Ni
\]

Notation:
- \( B_s \): Bainite Start Temperature [°C]
- **Alloy Amount**: [weight %]

Observations:
- Carbon Steels: \( r = 0.88 \), average error = 18°C. Useful range: \( 0.05 \leq C \leq 0.60\% \), \( 0.30 \leq Mn \leq 2.00 \), \( 0.15 \leq Si \leq 0.70\% \), \( Cr < 0.5\% \), \( 0.2\% \) Mo, \( 0.5\% \) Ni, \( 1 \leq ASTM \, GS \leq 10 \) and \( CR \geq 0.05°C/s \).
- Alloy Steels: \( r = 0.82 \), average error = 23°C. Useful range: \( 0.05 \leq C \leq 0.60\% \), \( 0.30 \leq Mn \leq 2.00 \), \( 0.15 \leq Si \leq 0.70\% \), \( Cr \leq 2.0\% \), \( Mo \leq 0.5\% \), \( Ni \leq 3.5\% \), \( 1 \leq ASTM \, GS \leq 10 \) and \( CR \geq 0.05°C/s \).


\[
B_s = 830 - 270 \, C - 90 \, Mn - 37 \, Ni - 70 \, Cr - 83 \, Mo
\]

\[
B_{50} = B_s - 50
\]

\[
B_{100} = B_s - 120
\]

Notation:
- \( B_s \): Bainite Start Temperature [°C]
- **Alloy Amount**: [weight %]
**B_s**: Temperature Required for the Formation of x% of Bainite [°C]

**Observations:**
- Equation determined using data from isothermal transformation diagrams.
- Reliable chemical composition range: 0.10-0.55% C, 0.2-1.7% Mn, 0.0-1.0% Mo, 0.0-3.5% Cr, 0.0-5.0% Ni.


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**. Suehiro**

\[ B_s = 718 - 425 C - 42.5 Mn \]

**Notation:**
- **B_s**: Bainite Start Temperature [°C]
- **Alloy Amount**: [weight %]


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**. Takada**

\[ B_s = 1336 - 1446 C - 62.3 Mn - 36.5 Si - 47.8 Cr - 160 V - 77.5 Mo \]

**Notation:**
- **B_s**: Bainite Start Temperature [K]
- **Alloy Amount**: [weight %]

**Observations:**
- Formula developed specifically for forging steels.
- Reliable chemical composition range: 0.11-0.40% C, 0.50-2.52% Mn, 0.31-1.26% Si, 0.20-1.96% Cr.


. Trzaska #1

\[ B_s = 675 - 212 C - 57 Mn - 17 Si - 29 Ni - 49 Cr - 60 Mo - 94 V + 0.056 T_A - 1.6 \frac{V}{v_R} \]

Notation:
- \( A_{r3} \): Ferrite Start Temperature [°C]
- **Alloy Content**: [weight %]
- \( T_A \): Austenitizing Temperature [°C]
- \( v_R \): Cooling Rate [°C/min]

Observations:
- Formula valid within the following range: 0.21% \( \leq C \leq 0.68\% \), 0.28% \( \leq Mn \leq 2.00\% \), 0.13% \( \leq Si \leq 1.90\% \), Cr \( \leq 2.5\% \), Ni \( \leq 3.85\% \), Mo \( \leq 1.05\% \), V \( \leq 0.38\% \) and Cu \( \leq 0.38\% \).
- Error = 30.6°C, \( r = 0.72 \).


. Trzaska #2

\[ B_s = 771 - 231.5 C - 69 Mn - 23 Si - 58.5 Cr - 31 Ni - 55 Mo - 41 V \]

Notation:
\[ B_s = 839 - 86 \, Mn - 23 \, Si - 67 \, Cr - 33 \, Ni - 75 \, Mo - 270 [1 - \exp(-1.33 \, C)] \]

Notation:
- **B\_s**: Bainite Start Temperature [°C]
- **Alloy Amount**: [weight %]

Observation:
- Factors multiplying substitutional elements are less than 10% different from the factors found by Steven and Haynes.
- Correlation Coefficient \( R^2 = 0.97 \), Standard Error of Estimate \( \sigma = 13 \)°C.


---

B\_s: Bainite Start Temperature [°C]

**Alloy Content**: [weight %]

Observations:
- Equation valid within the following alloy range: 0.06% \( \leq C \leq 0.68\%; 0.13 \leq Mn \leq 2.04\%; 0.12 \leq Si \leq 1.75\%; Ni \leq 3.85\%; Cr \leq 2.30\%; Mo \leq 1.05\%; V \leq 0.38\%; Cu \leq 0.38.
- Additional validity limitations: Mn + Cr \( \leq 3.6\); Mn + Cr + Ni \( \leq 5.6\); Cr + Ni \( \leq 5.3\); Mn + Ni \( \leq 4.5\).
- Regression coefficient \( r^2 = 0.75\); standard error: \( \pm 27.53\)°C.

\[ B_s = -36.6 \, Mn_{eq} \]

\[ Mn_{eq} = Mn + 3.43 \, Mo + 0.56 \, Ni \]

Notation:

- **Bₜ**: Bainite Start Temperature [°C]
- **Alloy Amount**: [weight %]


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**Zhao #1**

\[
B_s = 720 - 585.63 \, C + 126.60 \, C^2 - 91.68 \, Mn + 7.82 \, Mn^2 - 0.3378 \, Mn^3 - 66.34 \, Ni + 6.06 \, Ni^2 - 0.232 \, Ni^3 - 31.66 \, Cr + 2.17 \, Cr^2 - 42.37 \, Mo + 9.16 \, Co - 0.1255 \, Co^2 + 0.000284 \, Co^3 - 36.02 \, Cu - 46.15 \, Ru
\]


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**Zhao #2**

\[ B_s = 630 - 45 \, Mn - 40 \, V - 35 \, Si - 30 \, Cr - 25 \, Mo - 20 \, Ni - 15 \, W \]


Notation:
Bₜ: Bainite Start Temperature [°C]
Alloy Amount: [weight %]
- **Austenite Transformation Temperatures: Martensite Start and Finish**

- **. Andrews**

  **Linear**

  \[ M_s = 539 - 423C - 30.4\, Mn - 17.7\, Ni - 12.1\, Cr - 11.0\, Si - 7.5\, Mo \]

  **Non-Linear**

  \[ M_s = 512 - 453C - 16.9\, Ni - 9.5\, Mo + 217\, C^2 - 71.5\, C\, Mn + 15\, Cr - 67.6\, C\, Cr \]

  **Notation:**
  - **\( M_s \):** Martensite Start Temperature [°C]
  - **Alloy Content:** [weight %]

  **Observations:**
  - Formula valid for low alloy steels with less than 0.6%C, 4.9% Mn, 5.0% Cr, 5.0% Ni and 5.4% Mo.


- **. Capdevila**

  \[ M_s = 764.2 - 302.6\, C - 30.6\, Mn - 16.6\, Ni - 8.9\, Cr + 2.4\, Mo - 11.3\, Cu + 8.58\, Co + 7.4\, W - 14.5\, Si \]

  **Notation:**
  - **\( M_s \):** Martensite Start Temperature [K]
  - **Alloy Content:** [weight %]
Observations:
- Equation valid for steels with chemical composition between the following limits: $0.001 \leq C \leq 1.65$, $Mn \leq 3.76$, $Si \leq 3.40$, $Cr \leq 17.9$, $Ni \leq 27.2$, $Mo \leq 5.10$, $V \leq 4.55$, $Co \leq 30.0$, $Al \leq 1.10$, $W \leq 12.9$, $Cu \leq 0.98$, $Nb \leq 0.23$, $B \leq 0,0010$, $0.0001 \leq N \leq 0.060$.


. Carapella

\[
M_s = 496.1 \left(1 - 0.344 \, C\right) \left(1 - 0.051 \, Mn\right) \left(1 - 0.018 \, Si\right) \left(1 - 0.025 \, Ni\right) \left(1 - 0.039 \, Cr\right) \left(1 - 0.016 \, Mo\right) \left(1 - 0.010 \, W\right) \left(1 - 0.067 \, Co\right)
\]

Notation:
- $M_s$: Start Temperature of the Martensitic Transformation [°C]
- Alloy Amount: [weight %]

Source: CARAPELLA, L.A. Computing $A^{11}$ or $M_s$ (Transformation Temperature on Quenching), Metal Progress, 46, 1944, 108.

. Eichelman & Hull

\[
M_s = 41.7 \left(14.6 - Cr\right) + 5.6 \left(8.9 - Ni\right) + 33.3 \left(1.33 - Mn\right) + 27.8 \left(0.47 - Si\right) + 1666.7 \left(0.068 - C - N\right) - 17.8
\]

Notation:
- $M_s$: Martensite Start Temperature [°C]
- Alloy Content: [weight %]

Observations:
- Equation valid for 18-8 stainless steels.


**. Eldis**

\[ M_s = 531 - 391.2 \, C - 43.3 \, Mn - 21.8 \, Ni - 16.2 \, Cr \]

Notation:
- **M\textsubscript{s}:** Martensite Start Temperature [°C]
- **Alloy Content:** [weight %]

Observations:
- Equation valid for steels with chemical composition between the following limits: 0.10~0.80% C; 0.35~1.80% Mn; < 1.50% Si; < 0.90% Mo; < 1.50% Cr; < 4.50% Ni.


**. Finkler & Schirra**

\[ M_s = 635 - 474 \left[ C + 0.86 \left[ N - 0.15 \left( Nb + Zr \right) \right] - 0.066 \left( Ta + Hf \right) \right] - \left( 33 \, Mn + 17 \, Cr + 17 \, Ni + 21 \, Mo + 39 \, V + 11 \, W \right) \]

Notation:
- **M\textsubscript{s}:** Martensite Start Temperature [°C]
- **Alloy Content:** [weight %]
Observations:
- Equation valid for high temperature martensitic steels with 8,0 to 14% Cr.


**. Grange & Stewart**

\[ M_s = 537.8 - 361.1C - 38.9(Mn + Cr) - 19.4Ni - 27.8Mo \]

Notation:
- \( M_s \): Martensite Start Temperature [°C]
- **Alloy Amount**: [weight %]


**. Hougardy**

\[ M_s = 0.495M_{sjh} + 0.00095M_{sjh}^2 + 40 \]

Notation:
- \( M_s \): Corrected Martensite Start Temperature [°C]
- \( M_{sjh} \): Martensite Temperature Start According to Jaffe & Hollomon [°C]
- **Alloy Amount**: [weight %]

Observation:
- Correction of Jaffe & Hollomon equation considering several other similar equations already published.
\[ V_M = 1 - \exp[-k(M_s - T)^\delta] \]

\[ k = 0.36 \times 10^{-3} + 0.10 \times 10^{-4} M_s - 0.34 \times 10^{-6} M_s^2 + 0.32 \times 10^{-8} M_s^3 - 0.52 \times 10^{-11} M_s^4 \]

\[ q = 2.08 - 0.76 \times 10^{-2} M_s + 0.16 \times 10^{-2} M_s^2 - 0.90 \times 10^{-5} M_s^3 \]

Notation:
- \( V_M \): Volume Fraction of Martensite
- \( M_s \): Temperature at Which 1% Martensite Forms [°C]
- \( T \): Temperature [°C]


- Imai

\[ M_s = 539 - 423 C - 30.4 Mn - 7.5 Si + 30 Al \]

Notation:
- \( M_s \): Start Temperature of the Martensitic Transformation [°C]
- **Alloy Amount**: [weight %]

Observation:
- Formula specific for CMnAl TRIP steels.

. Jaffe & Hollomon

\[ M_s = 550 - 350 \, C - 40 \, Mn - 35 \, V - 20 \, Cr - 17 \, Ni - 10 \, Cu - 10 \, Mo - 8 \, W + 15 \, Co + 30 \, Al \]

Notation:
- \( M_s \): Start Temperature of the Martensitic Transformation [°C]
- **Alloy Amount**: [weight %]

Observation:
- Hougardy proposed a correction to this formula.


. Kunitake

\[ M_s = 560.5 - 407.3 \, C - 37.8 \, Mn - 14.8 \, Cr - 19.5 \, Ni - 4.5 \, Mo - 7.3 \, Si - 20.5 \, Cu \]

Notation:
- \( M_s \): Martensite Start Temperature [°C]
- **Alloy Amount**: [weight %]


. Lee & Park

\[ M_s = 475.9 - 335.1 \, C - 34.5 \, Mn - 1.3 \, Si - 15.5 \, Ni - 13.1 \, Cr - 10.7 \, Mo - 9.6 \, Cu + 11.67 \ln(d) \]
Notation:
- \( M_s \): Martensite Start Temperature [°C]
- \( d_v \): Austenite Grain Size [microns]
- **Alloy Amount**: [weight %]


- **Lee & Van Tyne**

\[
V_M = 1 - \exp[-K_{LV} (M_s - T)^{n_{LV}}]
\]

\[
K_{LV} = 0.0231 - 0.0105 C - 0.0017 Ni + 0.0074 Cr - 0.0193 Mo
\]

\[
n_{LV} = 1.4304 - 1.1836 C + 0.7527 C^2 + 0.0258 Ni - 0.0739 Cr + 0.3108 Mo
\]

Notation:
- \( V_M \): Volume Fraction of Martensite
- \( M_s \): Martensite Start Temperature [K]
- \( T \): Temperature [K]
- **Alloy Amount**: [weight %]

**Observation:**
- Start Temperature of Martensitic Transformation Calculated According to Capdevila.

. Li

\[ M_s = 540 - 420 \, C - 35 \, Mn - 12 \, Cr - 20 \, Ni - 21 \, Mo - 10.5 \, Si - 10.5 \, W + 20 \, Al + 140 \, V \]

Notation:
- \( M_s \): Martensite Start Temperature [°C]
- **Alloy Amount**: [weight %]


. Liu

\[ M_s = 550 - 361 \, C - 39 \, Mn - 35 \, V - 20 \, Cr - 17 \, Ni - 10 \, Cu - 5 \, Mo - 5 \, W + 16 \, Co + 30 \, Al \]

\[ M_s = 538 - 317 \, C - 33 \, Mn - 28 \, Cr - 17 \, Ni - 11 \, Si - 11 \, Mo - 11 \, W \]

. C < 0.05%

\[ M_s = 550 - 350 \, C - 45 \, Mn - 30 \, Cr - 20 \, Ni - 16 \, Mo - 5 \, Si - 8 \, W + 6 \, Co + 15 \, Al - 35 \, (V + Nb + Zr + Ti) \]

. C > 0.05%

\[ M_s = 525 - 350 \, (C - 0.05) - 45 \, Mn - 30 \, Cr - 20 \, Ni - 16 \, Mo - 5 \, Si - 8 \, W + 6 \, Co + 15 \, Al - 35 \, (V + Nb + Zr + Ti) \]

Notation:
- \( M_s \): Martensite Start Temperature [°C]
Alloy Amount: [weight %]


Lotter

\[ M_s = 558.3 - 312C - 49.0Mn - 24.9Cr - 649Ti \]

Observations:
- This formula was determined using data got from samples submitted to a normalizing rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; final cooling.
- $r = 0.94$; Standard Error of Deviation = 9.1°C

\[ M_s = 452.6 - 245C - 8.1Mn - 36.7Cr - 71.8Cu - 146.7Mo - 120V \]

Observations:
- This formula was determined using data got from samples submitted to a thermomechanical rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; 35% @800°C; final cooling.
- $r = 1.00$; Standard Error of Deviation = 1.5°C

Notation:
- $M_s$: Martensite Start Temperature [°C]
- Alloy Amount: [weight %]
- $\Delta t_{8/5}$: Time Between 800°C and 500°C [s]

. Mahieu

\[ M_s = 539 - 423 \ C - 30.4 \ Mn - 7.5 \ Si + 30.0 \ Al \]

Notation:
- \( M_s \): Martensite Start Temperature [°C]
- **Alloy Amount**: [weight %]

Observation:
- Equation valid for TRIP steels with 0.91 ≤ Al ≤ 1.73%. Apparently it is a development of the Andrews formula.


. Miettinen

Carbon Steels:

\[ M_s = 563.94 - 486.66 \ C + 56.48 \ S - 70.82 \ Mn + 21.77 \ C \ C - 33.52 \ C \ Si + 85.55 \ C \ Mn \]

Alloy Steels:

\[ M_s = 562.93 - 489.48 \ C + 59.74 \ Si - 72.37 \ Mn + 15.53 \ Cr - 26.96 \ Mo - 13.13 \ Ni + 25.90 \ C \ C - 39.43 \ C \ Si \\
\quad + 90.07 \ C \ Mn - 8.86 \ C \ Cr + 73.21 \ C \ Mo - 15.59 \ C \ Ni \]

Notation:
**Gorni**

*Steel Forming and Heat Treating Handbook*

**Ms**: Martensite Start Temperature [°C]

**Alloy Amount**: [weight %]

**Observations:**
- **Carbon Steels**: 
  \[ r = 0.99, \text{ average error} = 10^\circ C. \text{ Useful range: } 0.05 \leq C \leq 0.60\%, \ 0.30 \leq Mn \leq 2.00, \ 0.15 \leq Si \leq 0.70\%, \ Cr < 0.5\%, \ 0.2\% \text{ Mo}, \ 0.5\% \text{ Ni}, \ 1 \leq \text{ ASTM GS} \leq 10 \text{ and CR} \geq 0.05^\circ C/s. \]
- **Alloy Steels**: 
  \[ r = 0.98, \text{ average error} = 13^\circ C. \text{ Useful range: } 0.05 \leq C \leq 0.60\%, \ 0.30 \leq Mn \leq 2.00, \ 0.15 \leq Si \leq 0.70\%, \ Cr \leq 2.0\%, \ Mo \leq 0.5\%, \ Ni \leq 3.5\%, \ 1 \leq \text{ ASTM GS} \leq 10 \text{ and CR} \geq 0.05^\circ C/s. \]


---

. **Mikula & Wojnar**

\[
Ms = 635.02 - 549.82 \ C - 85.441 \ Mn - 68.967 \ Si - 18.07 \ Cr - 30.965 \ Ni - 69.301 \ Mo - 6.603 \ V + 420.26 \ Nb + 553.8 \ Ti - 1746.5 \ B
\]

**Notation:**
- **Ms**: Martensite Start Temperature [°C]
- **Alloy Amount**: [weight %]


---

. **Nehrenberg**

\[
Ms = 498.9 - 300 \ C - 33.3 \ Mn - 22.2 \ Cr - 16.7 \ Ni - 11.1 (Si + Mo)
\]

**Notation:**
- **Ms**: Martensite Start Temperature [°C]
Alloy Amount: [weight %]


Payson & Savage

\[ M_s = 498.9 - 316.7 \ C - 33.3 \ Mn - 27.8 \ Cr - 16.7 \ Ni - 11.1 (Si + Mo + W) \]

Notation:
- \( M_s \): Martensite Start Temperature [°C]
- Alloy Amount: [weight %]

Rowland & Lyle

\[ M_s[^\circ C] = 498.9 - 333.3 \ C - 33.3 \ Mn - 27.8 \ Cr - 16.7 \ Ni - 11.1 (Si + Mo + W) \]

\[ M_s[^\circ F] = 930 - 600 \ C - 60 \ Mn - 50 \ Cr - 30 \ Ni - 20 (Si + Mo + W) \]

\[ M_{10}[^\circ F] = M_s - 18 \]

\[ M_{30}[^\circ F] = M_s - 85 \]

\[ M_{90}[^\circ F] = M_s - 185 \]

\[ M_{100}[^\circ F] = M_s - 387 \]
Notation:
- $M_s$: Martensite Start Temperature
- $M_x$: Temperature Required for the Formation of $x\%$ of Martensite
- **Alloy Amount**: [weight %]


---

### Shimamura

$$M_s = 517 - 300 C - 11 Si - 33 Mn - 22 Cr - 17 Ni - 11 Mo$$

Notation:
- $M_s$: Martensite Start Temperature
- **Alloy Amount**: [weight %]


---

### Steven & Haynes

$$M_s = 561.1 - 473.9 C - 33 Mn - 16.7 (Cr + Ni) - 21.1 Mo$$

Notation:
- $M_s$: Martensite Start Temperature [°C]

. Sverdlin-Ness

\[ M_s = 520 - 320 C - 50 Mn - 30 Cr - 20 (Ni + Mo) - 5 (Cu + Si) \]

Notation:
- \( M_s \): Martensite Start Temperature [°C]
- **Alloy Amount**: [weight %]


. Tamura

\[ M_s = 550 - 361 C - 39 Mn - 20 Cr - 17 V - 17 Ni - 10 Cu - 5 (Mo + W) + 15 Co + 30 Al \]

Notation:
- \( M_s \): Martensite Start Temperature [°C]
- **Alloy Amount**: [weight %]


. Trzaska (I)

\[ M_s = 411 - 328 C - 13 Mn - 9 Ni - 3 Cr - 16 Mo + 34 Cu + 6.7 V_k^{0.25} \]

Notation:
- \( M_s \): Martensite Start Temperature [°C]
**Alloy Content:** [weight %]

**vr:** Cooling Rate [°C/min]

Observations:

- Formula valid within the following range: 0.21% ≤ C ≤ 0.68%, 0.28% ≤ Mn ≤ 2.00%, 0.13% ≤ Si ≤ 1.90%, Cr ≤ 2.5%, Ni ≤ 3.85%, Mo ≤ 1.05%, V ≤ 0.38% and Cu ≤ 0.38%.
- Error = 20.5°C, r = 0.88.


---

**M**$_s$ = 541 – 401 C – 36 Mn – 10.5 Si – 14 Cr – 18 Ni – 17 Mo

Notation:

- **M**$_s$: Martensite Start Temperature [°C]
- **Alloy Content:** [weight %]

Observations:

- Equation valid within the following alloy range: 0.06% ≤ C ≤ 0.68%; 0.13 ≤ Mn ≤ 2.04%; 0.12 ≤ Si ≤ 1.75%; Ni ≤ 3.85%; Cr ≤ 2.30%; Mo ≤ 1.05%; V ≤ 0.38%; Cu ≤ 0.38.
- Additional validity limitations: Mn + Cr ≤ 3.6; Mn + Cr + Ni ≤ 5.6; Cr + Ni ≤ 5.3; Mn + Ni ≤ 4.5
- Regression coefficient $r^2 = 0.87$; standard error $= ± 19.99°C$.


---

. van Bohemen
\[ f = 1 - \exp[-\alpha_m (T_{KM} - T)] \]

\[ T_{KM} = 462 - 273 \, C - 26 \, Mn - 13 \, Cr - 16 \, Ni - 30 \, Mo \]

\[ \alpha_m = 0.0224 - 0.0107 \, C - 0.0007 \, Mn - 0.00012 \, Cr - 0.00005 \, Ni - 0.0001 \, Mo \]

Notation:
- \( f \): Volume Fraction of Martensite as a Function of Undercooling Below \( T_{KM} \) Temperature (\( T_{KM} - T \))
- \( T \): Temperature [°C]
- \( T_{KM} \): Theoretical Martensite Start Temperature [°C]
- **Alloy Amount**: [weight %]

Observation:
- Formula based from Koistinen and Marburger equation.
- \( \alpha_m \): Standard error of estimate \( \sigma = 0.0014 \, K^{-1} \); correlation coefficient \( R^2 = 0.79 \).


\[ M_s = 565 - 31 \, Mn - 13 \, Si - 10 \, Cr - 18 \, Ni - 12 \, Mo - 600 \left[ 1 - \exp(-0.96 \, C) \right] \]

Notation:
- \( M_s \): Martensite Start Temperature [°C]
- **Alloy Amount**: [weight %]

Observation:
- Standard error of estimate \( \sigma = 13°C \); correlation coefficient \( r^2 = 0.95 \).
\[ f = 1 - \exp(-\alpha_m (M_s - T)) \]

\[ \alpha_m = 27.2 - 0.14 Mn - 0.21 Si - 0.11 Cr - 0.08 Ni - 0.05 Mo - 19.8 \left[ 1 - \exp(-1.56 C) \right] \]

Notation:
- \( f \): Volume Fraction of Martensite as a Function of Undercooling Below \( M_s \) Temperature (\( M_s - T \))
- \( T \): Temperature [°C]
- \( M_s \): Martensite Start Temperature [°C]
- \( \alpha_m \): Standard error of estimate \( \sigma = 0.005 \text{ K}^{-1}; \) correlation coefficient \( r^2 = 0.97 \).

Observation:
- Formula based from Koistinen and Marburger equation.
- \( \alpha_m \): Standard error of estimate \( \sigma = 0.005 \text{ K}^{-1}; \) correlation coefficient \( r^2 = 0.97 \).


---

\[ M_s^{TM} = 420 - 208.33 C - 33.428 Mn + 1.296 Mn^2 - 0.02167 Mn^3 - 16.08 Ni + 0.7817 Ni^2 - 0.02464 Ni^3 - 2.473 Cr + 30.00 Mo + 12.86 Co - 0.2654 Co^2 + 0.001547 Co^3 - 7.18 Cu - 72.65 N - 43.36 N^2 - 16.28 Ru + 1.72 Ru^2 - 0.08117 Ru^3 \]

\[ M_s^{LM} = 540 - 356.25 C - 47.59 Mn + 2.25 Mn^2 - 0.0415 Mn^3 - 24.56 Ni + 1.36 Ni^2 - 0.0384 Ni^3 - 17.82 Cr + 1.42 Cr^2 + 17.50 Mo + 21.87 Co - 0.468 Co^2 + 0.00296 Co^3 - 16.52 Cu - 260.64 N - 17.66 Ru \]

Notation:
- \( M_s^{TM} \): Twinned Martensite Start Temperature [°C]
- \( M_s^{LM} \): Lath Martensite Start Temperature [°C]
**Alloy Amount:** [weight %]

Observation:
- If $M_s^{LM}$ is higher than $M_s^{TM}$, both lath martensite and twinned martensite can be present in steel.
- However, if $M_s^{LM}$ is lower than $M_s^{TM}$, only twinned martensite can exist. This condition is fulfilled for some steels above a critical composition, which can be determined setting $M_s^{LM} = M_s^{TM}$.

Cooling Rate of Flat Products of Steel

. Bodnar

\[ \Delta t_{8/5} = 946 t^{-1.032} \]

Notation:
- \( \Delta t_{8/5} \): Average Cooling Rate of Steel Plate in Air Between 800°C and 500°C [°C/min]
- \( t \): Plate Thickness [mm]

Observation:
- Temperature evolution was measured at plate quarter-thickness.


. Degenkolbe

\[ CR_{8/5} = 4.52762 t^{-0.7494} \]

Notation:
- \( CR_{8/5} \): Cooling Rate in the Core of a Steel Plate in Air Between 800°C and 500°C [°C/s]
- \( t \): Plate Thickness [mm]

Observation:
- Equation fitted by the author of this review using data available in the figure 2 of the reference below.

- **Density of Bulk Steel at Ambient Temperature**

  . **Austenitic Steels**

  \[
  \rho_y = \frac{1}{(1.231Fe + 3.178C_{sol} + 1.307Mn + 2.436Si + 1.431Cr + 1.205Cu + 1.018Mo + 1.137Ni + 1.890Ti + 1.111Co + 2.186N + 2.032TiC).10^{-6}}
  \]

  Notation:
  - \(\rho_y\): Austenite Density [kg/m^3]
  - **Alloy/TiC Content**: [weight %]

  Observations:
  - \(C_{sol}\) is the content of this element not bound in TiC.
  - Density calculated at 20°C.


  . **Ferritic Steels**

  \[
  \rho_\alpha = \frac{1}{(1.270Fe + 1.380C + 1.524Mn + 2.381Si + 1.384Cr + 0.8477Cu + 1.076Mo + 1.370Ni + 2.012V + 4.046S).10^{-6}}
  \]

  Notation:
  - \(\rho_\alpha\): Ferrite Density [kg/m^3]
  - **Alloy Content**: [weight %]

  Observations:
  - C is considered insoluble in ferrite (that is, all C has gone to cementite).
  - The solubilities of the other alloy elements in cementite are zero.
  - Density calculated at 20°C.
Density of Fe-C Alloys in Heterogeneous Phase Mixtures

\[
\rho_{\text{Steel}} = \frac{1}{f_1 \rho_1 + f_2 \rho_2 + f_3 \rho_3 + \ldots + f_n \rho_n}
\]

Notation:
- \( \rho_{\text{Steel}} \): Steel Density [kg/m³]
- \( f_i \): Fraction of the phase \( i \) in the microstructure
- \( \rho_i \): Density of the phase \( i \)

## Density of Bulk Steel at High Temperature

**BISRA**

<table>
<thead>
<tr>
<th>T</th>
<th>P</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1008</td>
<td>1023</td>
<td>1040</td>
<td>1524</td>
</tr>
<tr>
<td>0</td>
<td>7861</td>
<td>7863</td>
<td>7858</td>
<td>7854</td>
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<tr>
<td>15</td>
<td>7856</td>
<td>7859</td>
<td>7854</td>
<td>7849</td>
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<td>50</td>
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<td>450</td>
<td>7711</td>
<td>7718</td>
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<td>500</td>
<td>7792</td>
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<td>7685</td>
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<tr>
<td>550</td>
<td>7673</td>
<td>7679</td>
<td>7672</td>
<td>7666</td>
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<td>600</td>
<td>7653</td>
<td>7659</td>
<td>7652</td>
<td>7646</td>
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<tr>
<td>650</td>
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<td>7635</td>
<td>7628</td>
<td>7622</td>
</tr>
<tr>
<td>700</td>
<td>7613</td>
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<td>750</td>
<td>7594</td>
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<td>800</td>
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</tr>
<tr>
<td>900</td>
<td>7600</td>
<td>7600</td>
<td>7590</td>
<td>7590</td>
</tr>
</tbody>
</table>
Notation:

ρ: Density of steel [kg/m³]
T: Temperature [°C]

Observations:
- Chemical composition of the steels [wt %]:

<table>
<thead>
<tr>
<th>Steel</th>
<th>C</th>
<th>Mn</th>
<th>Si</th>
<th>P</th>
<th>S</th>
<th>Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>1008</td>
<td>0.08</td>
<td>0.31</td>
<td>0.08</td>
<td>0.029</td>
<td>0.050</td>
<td>-</td>
</tr>
<tr>
<td>1023</td>
<td>0.23</td>
<td>0.64</td>
<td>0.11</td>
<td>0.034</td>
<td>0.034</td>
<td>0.13</td>
</tr>
<tr>
<td>1040</td>
<td>0.42</td>
<td>0.64</td>
<td>0.11</td>
<td>0.031</td>
<td>0.029</td>
<td>0.12</td>
</tr>
<tr>
<td>1524</td>
<td>0.23</td>
<td>1.51</td>
<td>0.12</td>
<td>0.037</td>
<td>0.038</td>
<td>0.11</td>
</tr>
</tbody>
</table>


. Picquè

\[ \rho = 7875.96 - 0.297T - 5.62 \times 10^{-5} T^2 \quad (T \leq \text{Ar}_3) \]

\[ \rho = 8099.79 + 0.506T \quad (T > \text{Ar}_3) \]

Notation:
\( \rho \): Density of steel \([\text{kg/m}^3]\)  
\( T \): Temperature \([\text{°C}]\)

Observation:
- Formulas specific for a 0.16% C, 0.5% Mn steel

- **Density of Liquid Steel**

  . **Jablonka**

  $\rho_{Steel} = (8319.49 - 0.835T) (1 - 0.01C)$

  Notation:
  
  $\rho_{Liquid Iron}$: Liquid Iron Density [kg/m³]
  
  $T$: Temperature, [°C]
  
  $C$: Carbon content [weight %]


  . **Yaws**

  $\rho_{Liquid Iron} = 1.9946 \times 0.22457 \left\{ \left[ 1 - \frac{T}{9340} \right]^{0.7} \right\}$

  Notation:
  
  $\rho_{Liquid Iron}$: Liquid Iron Density [g/ml]
  
  $T$: Absolute Temperature, [K]

- **Density of Microstructural Constituents at Ambient Temperature**

  . **Common Phases and Constituents**

<table>
<thead>
<tr>
<th>Phase/Constituent</th>
<th>C [weight %]</th>
<th>Specific Volume [cm³/g] at 20°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austenite</td>
<td>0.00 ~ 2.00</td>
<td>0.1212 + 0.0033 C</td>
</tr>
<tr>
<td>Martensite</td>
<td>0.00 ~ 2.00</td>
<td>0.1271 + 0.0025 C</td>
</tr>
<tr>
<td>Ferrite</td>
<td>0.00 ~ 0.02</td>
<td>0.1271</td>
</tr>
<tr>
<td>Cementite (Fe₃C)</td>
<td>6.7 ± 0.2</td>
<td>0.130 ± 0.001</td>
</tr>
<tr>
<td>ε Carbide</td>
<td>8.5 ± 0.7</td>
<td>0.140 ± 0.002</td>
</tr>
<tr>
<td>Graphite</td>
<td>100</td>
<td>0.451</td>
</tr>
<tr>
<td>Ferrite + Cementite</td>
<td>0.00 ~ 2.00</td>
<td>0.1271 + 0.0005 C</td>
</tr>
<tr>
<td>Low C Martensite + ε Carbide</td>
<td>0.25 ~ 2.00</td>
<td>0.1277 + 0.0015 (C − 0.25)</td>
</tr>
<tr>
<td>Ferrite + ε Carbide</td>
<td>0.00 ~ 2.00</td>
<td>0.1271 + 0.0015 C</td>
</tr>
</tbody>
</table>

Notation:
- **C**: Carbon Content [weight %]


- **Density and Molar Volume of Microalloy Carbides and Nitrides**

<table>
<thead>
<tr>
<th>Compound</th>
<th>Structure</th>
<th>Molecular Mass</th>
<th>Lattice Parameter [nm]</th>
<th>Molecules per Unit Cell</th>
<th>Density [g/cm³]</th>
<th>Molar Volume [cm³/mol]</th>
</tr>
</thead>
<tbody>
<tr>
<td>NbC</td>
<td>FCC</td>
<td>105</td>
<td>0.4462</td>
<td>4</td>
<td>7.84</td>
<td>13.39</td>
</tr>
<tr>
<td>NbN</td>
<td>FCC</td>
<td>107</td>
<td>0.4387</td>
<td>4</td>
<td>8.41</td>
<td>12.72</td>
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<tr>
<td>VC</td>
<td>FCC</td>
<td>63</td>
<td>0.4154</td>
<td>4</td>
<td>5.83</td>
<td>10.81</td>
</tr>
<tr>
<td>VN</td>
<td>FCC</td>
<td>65</td>
<td>0.4118</td>
<td>4</td>
<td>6.18</td>
<td>10.52</td>
</tr>
<tr>
<td>Compound</td>
<td>Density [kg/m³]</td>
<td>Molar Volume [cm³/mol]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>----------------</td>
<td>------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NbCN</td>
<td>9291</td>
<td>12.80</td>
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<td>ZrN</td>
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<tr>
<td>Mn</td>
<td>7470</td>
<td>7,35</td>
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</tr>
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<td>Si</td>
<td>2330</td>
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<td>Cr</td>
<td>7140</td>
<td>7,23</td>
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<td>Cu</td>
<td>8920</td>
<td>7,11</td>
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</tr>
<tr>
<td>C</td>
<td>2267</td>
<td>5,29</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>-</td>
<td>13,54</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td>7850</td>
<td>7.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations:
- Data based on room temperature lattice parameters.


**Density and Molar Volume of Precipitates and Metals**

<table>
<thead>
<tr>
<th>Compound</th>
<th>Density [kg/m³]</th>
<th>Molar Volume [cm³/mol]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TiC</td>
<td>60</td>
<td>0.4313</td>
</tr>
<tr>
<td>TiN</td>
<td>62</td>
<td>0.4233</td>
</tr>
<tr>
<td>AlN</td>
<td>CPH</td>
<td>41 c = 0.4965 a = 0.311</td>
</tr>
<tr>
<td>γ-Fe</td>
<td>FCC</td>
<td>56 0.357</td>
</tr>
<tr>
<td>α-Fe</td>
<td>BCC</td>
<td>56 0.286</td>
</tr>
</tbody>
</table>

Observations:
- Data based on room temperature lattice parameters.
Sources:


- Web Elements (www.webelements.com)


. Relationship Between Lattice Parameter and Density

\[
\rho = \frac{n \cdot M}{(a \cdot 10^{-10})^3 \cdot N}
\]

Notation:
- \( \rho \): Density [kg/m³]
- \( n \): Number of atoms per unit cell (depends on crystalline structure):
  - Cubic body-centered: 2
  - Cubic face-centered: 4
- \( M \): Molecular mass [kg/mol]:
  - Pure Fe: 0.055847
  - Cementite: 0.179552
- \( a \): Lattice Parameter [Å]
- \( N \): Avogadro’s number: 6.023 \( \cdot \) 10\(^{23} \)
- **Density of Microstructural Constituents at High Temperature**

  . **Fink**

  \[ \rho_y^T = \rho_y^{20\degree C} - 0.47T \]

  \[ \rho_a^T = \rho_a^{20\degree C} - 0.33T \]

  Notation:
  - \( \rho_y^T \): Austenite Density at Temperature T [kg/m³]
  - \( \rho_y^{20\degree C} \): Austenite Density at 20°C [kg/m³]
  - \( \rho_a^T \): Ferrite Density at Temperature T [kg/m³]
  - \( \rho_a^{20\degree C} \): Ferrite Density at 20°C [kg/m³]
  - \( T \): Temperature[°C]


  . **Jablonska**

  \[ \rho_y^T = (7875.96 - 0.297T - 5.62 \times 10^{-5} T^2 ) (1 - 2.62 \times 10^{-21} C) \]

  \[ \rho_y^T = (8099.79 - 0.506T) (1 - 1.46 \times 10^{-21} C) \]

  \[ \rho_a^T = (7875.96 - 0.297T - 5.62 \times 10^{-5} T^2 ) (1 - 2.62 \times 10^{-21} C) \]

  \[ \rho_{Fe3C}^T = (768645 - 6.63 \times 10^{-21} T - 3.12 \times 10^{-4} T^2 ) \]
Notation:
- $\rho_\delta^T$: Delta Ferrite Density at Temperature T [kg/m³]
- $\rho_\gamma^T$: Austenite Density at Temperature T [kg/m³]
- $\rho_\alpha^T$: Ferrite Density at Temperature T [kg/m³]
- $\rho_{Fe3C}^T$: Cementite Density at Temperature T [kg/m³]
- $T$: Temperature[°C]
- $C$: Carbon content [weight %]

Observations:
- Carbon content in ferrite is limited to 0.02% maximum.


**Molar Volume of Austenite as Function of Temperature**

$$V_M^\gamma = 6.688726 \times 10^{-6} \exp\left(7.3097 \times 10^{-5} \, T\right)$$

Notation:
- $V_M^\gamma$: Molar Volume of Austenite, [m³/mol]
- $T$: Temperature [K]


**Relationship Between Density and Thermal Expansion**
\[ \varepsilon^{th} = \sqrt{\frac{\rho(T_0)}{\rho(T)} - 1} \]

Notation:
- \( \varepsilon^{th} \): Thermal Expansion/Contraction
- \( \rho(T_0) \): Density at lower/higher \( T_0 \)
- \( \rho(T) \): Density at temperature \( T \)
- \( T_0 \): Reference temperature

- Dimensional Changes during Austenite Transformation

. During Cooling

\[
\frac{\Delta L_{\gamma\rightarrow\alpha}}{L} = 1.232 \times 10^{-2} - 1.347 \times 10^{-5} T - 6.544 \times 10^{-3} C_\gamma - 5.829 \times 10^{-3} C_\gamma^2 + 9.766 \times 10^{-6} C_\gamma T + 2.379 \times 10^{-6} T^2
\]

\[
C_\gamma = \frac{C_0 (1 - X_A) C_{tr}}{X_A}
\]

Notation:
- \(\Delta L_{\gamma\rightarrow\alpha}\): Specimen Length Change During Austenite Transformation [µm];
- \(L\): Specimen Original Length [µm]
- \(T\): Temperature [°C]
- \(C_\gamma\): Carbon Content in Austenite [wt %]
- \(C_0\): Initial Carbon Content [wt %]
- \(C_{tr}\): Carbon Content of Transformed Phases [wt %]
- \(X_A\): Untransformed Austenite Volume Fraction


. After General Heat Treating

<table>
<thead>
<tr>
<th>Transformation</th>
<th>(\Delta V) [%]</th>
<th>(\Delta l) [mm/mm]</th>
</tr>
</thead>
</table>


### Spheroidized Pearlite $\rightarrow$ Austenite

\[
-4.64 + 2.21 \text{ C} \quad -0.0155 + 0.0074 \text{ C}
\]

### Austenite $\rightarrow$ Martensite

\[
4.64 - 0.53 \text{ C} \quad 0.0155 - 0.0018 \text{ C}
\]

### Spheroidized Pearlite $\rightarrow$ Martensite

\[
1.68 \text{ C} \quad 0.0056 \text{ C}
\]

### Austenite $\rightarrow$ Lower Bainite

\[
4.64 - 1.43 \text{ C} \quad 0.0155 - 0.0048 \text{ C}
\]

### Spheroidized Pearlite $\rightarrow$ Lower Bainite

\[
0.78 \text{ C} \quad 0.0026 \text{ C}
\]

### Austenite $\rightarrow$ Upper Bainite

\[
4.14 - 2.21 \text{ C} \quad 0.0155 - 0.0074 \text{ C}
\]

### Spheroidized Pearlite $\rightarrow$ Upper Bainite

\[
0 \text{ (Zero)} \quad 0
\]

---

**Notation:**
- C: Carbon Content [weight %].

**Sources:**

### After Quenching

\[
\frac{\Delta V}{V} = \left( \frac{100 - V_C - V_A}{100} \right) 1.68 C_M + \frac{V_A}{100} (-4.64 + 2.21 C_A)
\]

**Notation:**
- $\Delta V/V$: Volumetric Change after Quenching [%]
- $V_C$: Non-solubilized Cementite Volumetric Fraction [%]
- $V_A$: Austenite Volumetric Fraction [%]
- $100 - V_C - V_A$: Martensite Volumetric Fraction [%]
- $C_M$: Carbon Content Solubilized in Martensite [weight %]
- $C_A$: Carbon Content Solubilized in Austenite [weight %]

- **Equivalent Carbon – H.A.Z. Hardenability**

  **. Dearden & O’Neill**

  \[
  C_{\text{EQ\_Dearden}} = C + \frac{Mn}{6} + \frac{Mo}{4} + \frac{Cr + V}{5} + \frac{Cu}{13} + \frac{Ni}{15} + \frac{P}{2}
  \]

  Notation:
  
  \( C_{\text{EQ\_Dearden}} \): Equivalent Carbon (Dearden) [%]
  
  **Alloy Content**: [weight %]


  **. Bastien**

  \[
  C_{\text{EQ\_Bastien}} = C + \frac{Mn}{4.4} + \frac{Mo}{7.7} + \frac{Cr}{15.4} + \frac{Ni}{10.3}
  \]

  \[
  \ln(CR_m) = 13.9 - 10.6 \times C_{\text{EQ\_Bastien}}
  \]

  Notation:
  
  \( C_{\text{EQ\_Bastien}} \): Equivalent Carbon (Bastien) [%]
  
  **Alloy Content**: [weight %]
  
  **CR\(m\)**: Critical Cooling Rate at 700°C [°C/s], that is, minimum cooling rate that produces a fully martensitic structure

. IIW - International Institute of Welding

\[
C_{\text{EQ}_{\text{IIW}}} = C + \frac{Mn}{6} + \frac{Cr + Mo + V}{5} + \frac{Cu + Ni}{15}
\]

Notation:
- \( C_{\text{EQ}_{\text{IIW}}} \): Equivalent Carbon (IIW) [%]
- **Alloy Content:** [weight %]

Source: HEISTERKAMP, F. et alii. Metallurgical Concept And Full-Scale Testing of High Toughness, \( H_2S \) Resistant 0.03\%C - 0.10\%Nb Steel. **C.B.M.M. Report**, São Paulo, February 1993.

. Kihara

\[
C_{\text{EQ}_{\text{Kihara}}} = C + \frac{Mn}{6} + \frac{Mo}{4} + \frac{Cr}{5} + \frac{V}{14} + \frac{Ni}{40} + \frac{Si}{24}
\]

Notation:
- \( C_{\text{EQ}_{\text{Kihara}}} \): Equivalent Carbon (Kihara) [%]
- **Alloy Content:** [weight %]


. Shinozaki

\[
C_{\text{EQ}_{\text{FBW}}} = C + \frac{Mn}{5} + \frac{Si}{15} + \frac{Cr}{9} + 7 \text{Nb} (1 - 10C) + \frac{V (50C - 1)}{3} + 1.3 \text{Ti} (1 - 5C) + \frac{Mo (1 - 6C)}{2} + 29 \text{B} (11C - 1)
\]

Notation:
**Ceq_FBW**: Equivalent Carbon Designed Specifically for Flash Butt Welding [%]

**Alloy Content**: [weight %]


### Stout

\[
C_{EQ\_Stout} = 1000 \left( C + \frac{Mn}{6} + \frac{Cr + Mo}{10} + \frac{Ni}{20} + \frac{Cu}{40} \right)
\]

Notation:

\(C_{EQ\_Stout}\): Equivalent Carbon (Kihara) [%]

**Alloy Content**: [weight %]


### Yurioka

\[
C_{EQ\_Yurioka} = C + \frac{Mn}{6} + \frac{Mo}{4} + \frac{Cr}{8} + \frac{Ni}{12} + \frac{Si}{24} + \frac{Cu}{15}
\]

\[
\log(t_m) = 10.6 C_{EQ\_Yurioka} - 4.8
\]

Notation:

\(C_{EQ\_Yurioka}\): Equivalent Carbon (Yurioka) [%]

**Alloy Content**: [weight %]

**tm**: Critical Cooling Time from 800 to 500°C [s] (that is, maximum cooling time that produces a fully martensitic structure)
- **Equivalent Carbon – Hydrogen Assisted Cold Cracking**

  . Bersch & Koch (Hoesch)

  \[
  C_{EQ, Bersch} = C + \frac{Mn + Si + Cr + Mo + V + Cu + Ni}{20}
  \]

  Notation:
  - \( C_{EQ, Bersch} \): Equivalent Carbon for Pipeline Steels [%]
  - **Alloy Content**: [weight %]

  Observations:
  - Formula deduced for pipeline steels

  Source:


  . DNV

  \[
  C_{EQ, DNV} = C + \frac{Mn}{10} + \frac{Si}{24} + \frac{Ni + Cu}{40} + \frac{Cr}{5} + \frac{V}{10} + \frac{Mo}{4}
  \]

  Notation:
  - \( C_{EQ, DNV} \): Equivalent Carbon (DNV) [%]
  - **Alloy Content**: [weight %]

\[ \text{Graville} \]

\[
C_{\text{EQ,HSLA}} = C + \frac{Mn}{16} - \frac{Ni}{50} + \frac{Cr}{23} + \frac{Mo}{7} + \frac{Nb}{5} + \frac{V}{9}
\]

Notation:
- \( C_{\text{EQ,HSLA}} \): Equivalent Carbon (Uwer & Graville) [%]
- **Alloy Content**: [weight %]

Observations:
- Formula deduced for pipeline steels


\[ \text{Ito & Bessyo (I)} \]

\[
P_{\text{cm}} = C + \frac{Si}{30} + \frac{Mn + Cu + Cr}{20} + \frac{Ni}{60} + \frac{Mo}{15} + \frac{V}{10} + 5B
\]

Notation:
- \( P_{\text{cm}} \): Cracking Parameter [%]
- **Alloy Content**: [weight %]

Observations:
- Formula deduced for pipeline steels with \( C < 0.15\% \)
- This is the most popular formula for this kind of material.
- Equation valid under the following conditions: $0.07\% \leq C \leq 0.22\%$, $0.40\% \leq Mn \leq 1.40\%$, $Si \leq 0.60\%$, $V \leq 0.12\%$, $Cr \leq 1.20\%$, $Ni \leq 1.20\%$, $Cu \leq 0.50\%$, $Mo \leq 0.7\%$, $B \leq 0.005\%$.

Sources:


. **Ito & Bessyo (II)**

\[
P_c = C + \frac{Si}{30} + \frac{Mn + Cu + Cr}{20} + \frac{Mo}{15} + \frac{V}{10} + \frac{d}{600} + \frac{H}{60}
\]

Notation:
- $P_c$: Cracking Parameter [%]
- **Alloy Content**: [weight %], except
- $H$: Hydrogen amount in the weld metal, [cm³/100 g]
- $d$: Plate Thickness, [mm]


. **Mannesmann**

\[
C_{EQ_{PLS}} = C + \frac{Si}{25} + \frac{Mn + Cu}{16} + \frac{Cr}{20} + \frac{Ni}{60} + \frac{Mo}{40} + \frac{V}{15}
\]

Notation:
**C_{EQ,PLS}:** Equivalent Carbon for Pipeline Steels [%]

**Alloy Content:** [weight %]

**Observations:**
- Formula deduced for pipeline steels
- A version of this formula divides $V$ by 10

**Sources:**
- HEISTERKAMP, F. et alii.: **Metallurgical Concept And Full-Scale Testing of High Toughness, H$_2$S Resistant 0.03%C - 0.10%Nb Steel. C.B.M.M. Report**, São Paulo, February 1993.

**. Uwer & Hohne**

$$C_{EQ,Uwer} = C + \frac{Mn}{10} + \frac{Cu}{20} + \frac{Ni}{40} + \frac{Cr}{20} + \frac{Mo}{10}$$

**Notation:**
- $C_{EQ,Uwe}$: Equivalent Carbon (Uwer & Hohne) [%]
- **Alloy Content:** [weight %]


**. Yurioka**
\[ C_{EQ_{Yurioka}} = C + A(C) \left( \frac{Mn}{6} + \frac{Si}{24} + \frac{Cr + Mo + V}{5} + \frac{Cu}{15} + \frac{Ni}{20} + \frac{Nb}{5} + 5 B \right) \]

\[ A(C) = 0.75 + 0.25 \tanh \left[ 20 (C - 0.12) \right] \]

Notation:
- \( C_{EQ_{Yurioka}} \): Equivalent Carbon for Pipeline Steels [%]
- \textbf{Alloy Content}: [weight %]

Observations:
- Formula for C-Mn and microalloyed pipeline steels
- This formula combines Carbon Equivalent equations from IIW and \( P_{cm} \)

Sources:

- **Equivalent Carbon – Peritectic Point**
  
  **Blazek**

\[
C_A = 0.0896 + 0.0458\, Al - 0.0205\, Mn - 0.0077\, Si - 0.0223\, Al^2 - 0.0239\, Ni + 0.0106\, Mo + 0.0134\, V - 0.0032\, Cr + 0.00059\, Cr^2 + 0.0197\, W
\]

\[
C_B = 0.1967 + 0.036\, Al - 0.0316\, Mn - 0.0103\, Si + 0.1411\, Al^2 + 0.05\, Al\, Si - 0.0401\, Ni + 0.03255\, Mo + 0.0603\, V + 0.0024\, Cr + 0.00142\, Cr^2 - 0.00059\, Cr\, Ni + 0.0266\, W
\]

**Notation:**
- **C_A**: Maximum Carbon Solubility of the δ-Ferrite Phase [%].
- **C_B**: Peritectic composition of the Fe-C phase diagram [%].
- **Alloy Content**: [weight %]

**Observations:**
- Formula valid within the following ranges: Al \(<\) 2.0\%, Cr \(<\) 18.3\%, Mn \(<\) 2.1\%, Mo \(<\) 2.2\%, Si \(<\) 2.05\%, P \(<\) 0.1\%, S \(<\) 0.15\%, Cu \(<\) 1.35\%, Ni \(<\) 10.3\%, V \(<\) 1.08\%, Ti \(<\) 0.22\%, Sn \(<\) 0.03\%, Nb \(<\) 0.075\% and W \(<\) 0.5\%.
- C_A: \(r^2 = 0.99\), RMS Error = 0.0053\%.
- C_B: \(r^2 = 0.98\), RMS Error = 0.0126\%.
- This model fitted data generated by the Thermocalc software.


**Mills**

\[
C_{\text{P-Mills}} = C + 0.02\, Mn + 0.04\, Ni - 0.1\, Si - 0.04\, Cr - 0.1\, Mo
\]

**Notation:**
**C\textsubscript{P} \textsubscript{Mills}:** Equivalent Carbon for Peritectic Point [%]

**Alloy Content:** [weight %]


\[ C_{P_{\text{Inf}}} = f_1 - 0.10 \]
\[ C_{P_{\text{Sup}}} = f_2 + 0.05 \]

\[ f_1 = 0.0828 \text{Si} - 0.0195 \text{Mn} + 0.07398 \text{Al} - 0.04614 \text{Ni} + 0.02447 \text{Cr} + 0.01851 \text{Mo} + 0.090 \]
\[ f_2 = 0.2187 \text{Si} - 0.03291 \text{Mn} + 0.2017 \text{Al} - 0.06715 \text{Ni} + 0.04776 \text{Cr} + 0.04601 \text{Mo} + 0.173 \]

Notation:
- **C\textsubscript{P}\textsubscript{Inf}:** Lower Bound of Carbon Peritectic Content Range [%]
- **C\textsubscript{P}\textsubscript{Sup}:** Upper Bound of Carbon Peritectic Content Range [%]
- **Alloy Content:** [weight %]

Observations:
- Alloy elements contents are assumed to be 4.0% or less, excluding 0%.


**. Shepherd**
\[ C'_\delta = 0.0927 - 0.0151 \text{Mn} + 0.00776 \text{Si}^2 + 0.0565 \text{Al} + 0.0143 \text{Al}^2 + 0.00338 \text{Al}^3 - 0.0170 \text{Mn Si} - 0.0148 \text{Mn Al} - 0.0574 \text{Si Al} - \\
- 0.00848 \text{Mn Si Al} - 0.00900 (\text{Si Al})^2 - 0.0121 (\text{Si + Al}) - 0.000775 \text{Si}^4 + 0.00128 (\text{Mn Si})^3 + 0.00119 (\text{Mn + Si + Al})^3 + \\
+ 0.000913 (\text{Mn Al})^4 - 0.00193 (\text{Mn Si Al})^4 - 0.000341 (\text{Mn + Si + Al})^4 - 0.0425 P^2 + 0.0549 P + 0.1369 S - 0.0135 \text{Cu} - \\
- 0.4694 N + 0.0036 \text{Sn}^2 - 0.014 \text{Sn} - 0.0256 \text{Nb} - 0.0357 \text{Ti} + 0.0113 V - 0.0009 \text{Mo}^2 + 0.0062 \text{Mo} - 0.0016 \text{Cr} - \\
- 0.0195 \text{Ni} \\
\]

\[ C'_\gamma = 0.249 + 0.0673 \text{Si}^2 + 0.177 \text{Al}^2 - 0.0232 \text{Mn Si} - 0.0116 \text{Mn Al} + 0.140 \text{Al}^3 - 0.105 \text{Si Al} + 0.0214 \text{Mn Si Al} + \\
+ 0.0104 (\text{Mn Si})^2 - 0.0429 (\text{Si Al})^2 - 0.195 \text{Al}^4 + 0.0441 \text{Mn}^4 - 0.0269 \text{Mn}^5 - 0.0242 \text{e}^{\text{Mn}} - 0.0437 \text{e}^{\text{Si}} + 0.0233 (\text{Si Al})^4 + \\
+ 0.0152 (\text{Mn Si Al})^4 - 0.000721 (\text{Mn + Si + Al})^4 + 0.2651 P + 0.5573 S - 0.0174 \text{Cu} - 0.585 N + 0.0094 \text{Sn}^2 - 0.0211 \text{Sn} - \\
- 0.027 \text{Nb} + 0.0377 \text{Ti}^2 - 0.0463 \text{Ti} + 0.042 V - 0.0015 \text{Mo}^2 + 0.0238 \text{Mo} + 0.0024 \text{Cr}^2 - 0.002 \text{Cr} - 0.0349 \text{Ni} \\
\]

\[ C'_{\text{liq}} = 0.746 - 0.0469 \text{Mn} + 0.0305 \text{Si} - 0.0265 \text{Si}^2 + 0.0236 \text{Si}^3 + 1.37 \text{Al} - 1.21 \text{Al}^2 + 1.70 \text{Al}^3 - 0.771 \sqrt{\text{Al}} + \\
+ 0.0745 \log_{10}(\text{Al}) - 0.0351 \text{Mn Si} - 0.0560 \text{Mn Al} - 0.249 \text{Si Al} + 0.00571 \text{Mn Si Al} - 0.00973 (\text{Mn·Al})^3 - 1.07 \text{Al}^4 + \\
+ 0.321 \text{Al}^5 + 0.00544 (\text{Mn + Al})^2 - 0.0338 (\text{Si Al})^3 + 0.1065 P + 1.239 S - 0.0621 \text{Cu} - 0.8642 N + 0.001 \text{Sn}^2 - \\
- 0.0191 \text{Sn} + 0.0051 \text{Ti}^2 + 0.0386 \text{Ti} + 0.0043 V^2 + 0.0896 V + 0.046 \text{Mo} + 0.0056 \text{Cr}^2 - 0.0178 \text{Cr} - \\
- 0.0047 \text{Ni}^2 - 0.0763 \text{Ni} \\
\]

**Notation:**

- \( C'_\delta \): Maximum Carbon Solubility of the \( \delta \)-Ferrite Phase [%]
- \( C'_\gamma \): Peritectic Composition [%]
- \( C'_{\text{liq}} \): Maximum Carbon Solubility of the Liquid Phase [%]

**Alloy Content:** [weight %]

**Observations:**
- Range of Validity: Mn ≤ 2.0%, P ≤ 0.192%, S ≤ 0.070%, Si ≤ 1.5%, Cu ≤ 1.0%, Al ≤ 2.0%, N ≤ 0.1%, Sn ≤ 1.0%, Nb ≤ 0.5%, Ti ≤ 0.44%, V ≤ 1.0%, Mo ≤ 1.0%, Cr ≤ 0.5% and Ni ≤ 0.5%.
- Standard Deviation: C′_δ = 0.0002, C′_δ = 0.0011, C′_{liq} = 0.0023.
- This model fitted data generated by the FactSage software.


**. Wolf**

\[ C_{P_{WOLF}} = C + 0.04 \text{Mn} + 0.1 \text{Ni} + 0.7 N - 0.14 Si - 0.04 Cr - 0.1 Mo - 0.4 Ti \]

Notation:
- \( C_{P_{WOLF}} \): Equivalent Carbon for Peritectic Point [%]
- **Alloy Content**: [weight %]


**. Xu**

\[ C_{P_{XU}} = 0.1763 + 0.0616 Al - 2.5275 S + 0.2652 P - 0.0023 Si - 0.0344 Mn + 1.5250 S Mn - 0.0210 S Mn \]

Notation:
- \( C_{P_{XU}} \): Equivalent Carbon for Peritectic Point [%]
- **Alloy Content**: [weight %]

Observations:
- Formula valid within the following ranges: Si ≤ 0.5%, Mn ≤ 1.5%, Al ≤ 0.06%, P ≤ 0.05%, S ≤ 0.015%.
- $r^2 = 0.978$
- Fitting data generated by FactSage software.

- Fe-C Equilibrium Diagram

Source: ASM's Heat Treating One-Minute Mentor,
http://www.asminternational.org/pdf/HTSRefCharts/Vol4p4Fig1.pdf.
- **Fe-C Equilibrium Equations in the Solidification and Eutectoid Range**

  . **Liquids of \( \delta \)-iron**

  \[
  \%C = \frac{1536 - T}{79}
  \]

  . **Liquids of \( \gamma \)-iron**

  \[
  \%C = \frac{1525 - T - 1.1284 \times 10^{-3} (T - 1525)^2}{56.0198} - \frac{56.0198}{56.0198}
  \]

  Notation:
  
  \( \%C \): coordinate in the Fe-C diagram [%]
  
  \( T \): Temperature [°C]

  . **Solidus of \( \delta \)-iron**

  \[
  \%C = \frac{1536 - T}{460}
  \]

  . **Solidus of \( \gamma \)-iron**

  \[
  \%C = \frac{1525 - T}{185}
  \]
. Start of transformation of $\delta$-iron (on cooling)

$$\% C = \frac{T - 1392}{1140}$$

. End of transformation of $\delta$-iron (on cooling)

$$\% C = \frac{T - 1392}{624.3749}$$

. $A_3$ Line

$$\% C = \frac{911 - T}{484.785} \cdot \frac{2.818 \times 10^{-4} \cdot (T - 911)^2}{484.785} - \frac{2.8574 \times 10^{-5} \cdot (T - 911)^3}{484.785}$$

. $A_{cm}$ Line

$$\% C = 0.8 + \frac{T - 723}{453.7137} + \frac{7.7917 \times 10^{-4} \cdot (T - 723)^2}{453.7137}$$

- **Ferrite Solubility Products**

  . **General**

\[
\log_{10} \frac{(a_\alpha)^m (a_\beta)^n}{a_{A_m B_n}} = - \frac{A}{T} + B
\]

Notation:
- \(A_m B_n\): Precipitate Considered for Calculation
- \(a_\alpha\): Alloy Content [weight %]
- \(T\): Temperature [K]
- \(A, B\): Constants of the Solubility Product, given in the table below:

<table>
<thead>
<tr>
<th>Precipitate</th>
<th>A</th>
<th>B</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlN</td>
<td>9595</td>
<td>2.65</td>
<td>Kunze &amp; Reichert</td>
</tr>
<tr>
<td>BN</td>
<td>13560</td>
<td>4.53</td>
<td>Fountain &amp; Chipman</td>
</tr>
<tr>
<td>MnS</td>
<td>8400</td>
<td>2.77</td>
<td>Ivanov</td>
</tr>
<tr>
<td>NbC</td>
<td>10990</td>
<td>4.62</td>
<td>Kunze</td>
</tr>
<tr>
<td>NbN</td>
<td>10650</td>
<td>3.87</td>
<td>Kunze</td>
</tr>
<tr>
<td>TiN</td>
<td>17640</td>
<td>6.17</td>
<td>Kunze</td>
</tr>
<tr>
<td>VC</td>
<td>12265</td>
<td>8.05</td>
<td>Taylor</td>
</tr>
<tr>
<td>VN</td>
<td>7830</td>
<td>2.45</td>
<td>Froberg</td>
</tr>
<tr>
<td>VN</td>
<td>8120</td>
<td>2.48</td>
<td>Roberts &amp; Sandbert</td>
</tr>
<tr>
<td>ZrN</td>
<td>18160</td>
<td>5.24</td>
<td>Kunze</td>
</tr>
</tbody>
</table>

**Observations:**
- \(a_{\text{AmBn}}\) is equal to one if the precipitate is pure.
- \(a_{\text{AmBn}} \leq 1\) if there is co-precipitation with another element.

Sources:

- **Hardness After Austenite Cooling**
  
  **Blondeau**

\[
HV = f_{FP} HV_{FP} + f_B HV_B + f_M HV_M
\]

\[
HV_{FP} = 42 + 223 \text{ C} + 53 \text{ Si} + 30 \text{ Mn} + 7 \text{ Cr} + 19 \text{ Mo} + 126 \text{ Ni} + (10 - 19 \text{ Si} + 8 \text{ Cr} + 4 \text{ Ni} + 130 \text{ V}) \log_{10}(v_r)
\]

\[
HV_B = -323 + 185 \text{ C} + 330 \text{ Si} + 153 \text{ Mn} + 144 \text{ Cr} + 191 \text{ Mo} + 65 \text{ Ni} + (89 + 53 \text{ C} - 55 \text{ Si} - 22 \text{ Mn} - 20 \text{ Cr} - 33 \text{ Mo} - 10 \text{ N}) \log_{10}(v_r)
\]

\[
HV_M = 127 + 949 \text{ C} + 27 \text{ Si} + 11 \text{ Mn} + 16 \text{ Cr} + 8 \text{ Ni} + 21 \log_{10}(v_f)
\]

\[
\log(v_M) = 9.81 \left( -4.62 \text{ C} + 1.05 \text{ Mn} + 0.54 \text{ Ni} + 0.50 \text{ Cr} + 0.66 \text{ Mo} + 0.00183 \text{ PA} \right)
\]

\[
\log(v_{M90}) = 8.76 - (4.04 \text{ C} + 0.96 \text{ Mn} + 0.58 \text{ Cr} + 0.97 \text{ Mo} + 0.49 \text{ Ni} + 0.001 \text{ PA})
\]

\[
\log(v_{M50}) = 8.50 - (4.13 \text{ C} + 0.86 \text{ Mn} + 0.41 \text{ Cr} + 0.94 \text{ Mo} + 0.57 \text{ Ni} + 0.0012 \text{ PA})
\]

\[
\log(v_B) = 10.17 - (3.80 \text{ C} + 1.07 \text{ Mn} + 0.70 \text{ Ni} + 0.57 \text{ Cr} + 1.58 \text{ Mo} + 0.0032 \text{ PA})
\]

\[
\log(v_{B90}) = 10.55 - (3.65 \text{ C} + 1.08 \text{ Mn} + 0.61 \text{ Cr} + 1.49 \text{ Mo} + 0.77 \text{ Ni} + 0.0032 \text{ PA})
\]

\[
\log(v_{B50}) = 8.74 - (2.23 \text{ C} + 0.86 \text{ Mn} + 0.59 \text{ Cr} + 1.60 \text{ Mo} + 0.56 \text{ Ni} + 0.032 \text{ PA})
\]

\[
\log(v_{FP}) = 6.36 - (0.43 \text{ C} + 0.49 \text{ Mn} + 0.78 \text{ Ni} + 0.27 \text{ Cr} + 0.38 \text{ Mo} + 2 \sqrt{\text{Mo}} + 0.0019 \text{ PA})
\]

\[
\log(v_{FP90}) = 7.51 - (1.38 \text{ C} + 0.35 \text{ Mn} + 0.11 \text{ Cr} + 2.31 \text{ Mo} + 0.93 \text{ Ni} + 0.0033 \text{ PA})
\]
\[ PA = \left[ \frac{1}{T} - \frac{4.58 \log_{10}(t)}{\Delta H} \right]^{-1} \]

**Notation:**
- \textbf{HV}: Global Hardness [Vickers]
- \textbf{f}_{FP}: Fraction of Ferrite-Pearlite in Microstructure
- \textbf{f}_{B}: Fraction of Bainite in Microstructure
- \textbf{f}_{M}: Fraction of Martensite in Microstructure
- \textbf{HV}_{FP}: Hardness of Ferrite-Pearlite [Vickers]
- \textbf{HV}_{B}: Hardness of Bainite [Vickers]
- \textbf{HV}_{M}: Hardness of Martensite [Vickers]
- \textbf{Alloy Content}: [weight %]
- \textbf{v}_{r}: Applied Cooling Rate at 700°C [°C/h]
- \textbf{v}_{M}: Critical Cooling Rate at 700°C for Martensitic Quenching [°C/h]
- \textbf{v}_{M90}: Cooling Rate at 700°C Required for 90% Martensite [°C/h]
- \textbf{v}_{M50}: Cooling Rate at 700°C Required for 50% Martensite [°C/h]
- \textbf{v}_{B}: Critical Cooling Rate at 700°C for Bainitic Quenching [°C/h]
- \textbf{v}_{B90}: Cooling Rate at 700°C Required for 90% Bainite [°C/h]
- \textbf{v}_{B50}: Cooling Rate at 700°C Required for 50% Bainite [°C/h]
- \textbf{v}_{FP}: Critical Cooling Rate at 700°C for Annealing (Ferrite-Pearlite) [°C/h]
- \textbf{v}_{FP90}: Cooling Rate at 700°C Required for 90% Ferrite-Pearlite [°C/h]
- \textbf{PA}: Austenitization Parameter [K]
- \textbf{T}: Austenitization Temperature [K]
- \textbf{t}: Austenitization Soaking Time [h]
- \textbf{ΔH}: Austenitization Activation Energy: 240 kJ/mol for C Steels; 418 kJ/mol if Mo ≥ 0.04%, 461 kJ/mol for most alloy steels

**Observations:**
- Limits for Austenitization: 1073 K (800°C) ≤ T ≤ 1373 K (1100°C) and t ≤ 1 h.
- Equations Valid for the Following Chemical Composition Range: 0.10% < C < 0.50%, Mn < 2.0%, Si < 1.0%, Ni ≤ 4.0%, Cr < 3.0%, Mo < 1.0%, Cu < 0.5%, V < 0.2%, 0.010% < Al < 0.050% and Mn + Ni + Cr + Mo < 5.0%.

. Lorenz

\[HV = 2019 \left[ C \left(1 - 0.5 \log t_{8/5}\right) + 0.3 \left(\frac{Si}{11} + \frac{Mn}{8} + \frac{Cu}{9} + \frac{Cr}{5} + \frac{Ni}{17} + \frac{Mo}{6} + \frac{V}{3}\right)\right] + 66 \left(1 - 0.8 \log t_{8/5}\right)\]

Notation:

HV: Maximum Hardness for a Martensitic-Bainitic HAZ Microstructure [Vickers, 10 kg Load]
Alloy Content: [weight %]
t_{8/5}: Cooling Time Between 800°C and 500°C [s]


. Lotter

\[\Delta t_{8/5} = 2 \text{ s}\]

\[HV_{10} = 177.7 + 975.4 \, C + 42.4 \, Mn + 163.3 \, Mo - 1247 \, Nb\]

- \( r = 0.89; \) Root Mean Square Deviation = 54 \( HV_{10} \)

\[\Delta t_{8/5} = 10 \text{ s}\]

\[HV_{10} = 103.6 + 1094.5 \, C + 346.0 \, Mo\]
- $r = 0.91$; Root Mean Square Deviation = 49 HV$_{10}$

$\Delta t_{s/5} = 50$ s

$HV_{10} = 86.8 + 341.5 \, C + 52.0 \, Mn - 85.9 \, Si + 152.2 \, Cr + 275.9 \, Mo$

- $r = 0.88$; Root Mean Square Deviation = 37 HV$_{10}$

$\Delta t_{s/5} = 100$ s

$HV_{10} = 79.5 + 317.5 \, C + 57.4 \, Mn - 107.1 \, Si + 141.0 \, Cr + 225.9 \, Mo$

- $r = 0.88$; Root Mean Square Deviation = 33 HV$_{10}$

$\Delta t_{s/5} = 500$ s

$HV_{10} = 61.7 + 277.9 \, C + 34.9 \, Mn + 94.7 \, Cr + 96.7 \, Cu + 165.1 \, Mo$

- $r = 0.89$; Root Mean Square Deviation = 27 HV$_{10}$

$\Delta t_{s/5} = 1000$ s

$HV_{10} = 56.6 + 262.6 \, C + 33.7 \, Mn + 74.9 \, Cr + 121.8 \, Cu + 155.9 \, Mo$

- $r = 0.88$; Root Mean Square Deviation = 26 HV$_{10}$

Observations:
- These formulas were determined using data got from samples submitted to a normalizing rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition. Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; final cooling.
\[ \Delta t_{8/5} = 2 \text{ s} \]

\[ HV_{10} = 107.3 + 1107.5 C + 47.4 Mn + 60.9 Ni + 204.9 Mo - 22856.0 B \]

- \( r = 0.97 \); Root Mean Square Deviation = 32 HV_{10}

\[ \Delta t_{8/5} = 10 \text{ s} \]

\[ HV_{10} = 84.2 + 773.5 C + 43.6 Mn + 106.5 Cr + 192.3 Mo + 28510.0 B \]

- \( r = 0.92 \); Root Mean Square Deviation = 40 HV_{10}

\[ \Delta t_{8/5} = 50 \text{ s} \]

\[ HV_{10} = 36.6 + 398.0 C + 81.6 Mn - 138.1 Si + 191.7 Cr + 246.7 Mo + 302.0 Ti + 1018.0 Nb \]

- \( r = 0.95 \); Root Mean Square Deviation = 23 HV_{10}

\[ \Delta t_{8/5} = 100 \text{ s} \]

\[ HV_{10} = 25.4 + 371.2 C + 77.7 Mn - 103.9 Si + 186.8 Cr + 216.7 Mo + 241.0 Ti + 1095.0 Nb \]

- \( r = 0.96 \); Root Mean Square Deviation = 19 HV_{10}

\[ \Delta t_{8/5} = 500 \text{ s} \]

\[ HV_{10} = 25.7 + 318.8 C + 61.0 Mn - 60.4 Si + 135.3 Cr + 70.1 Cu + 201.9 Mo + 224.0 Ti + 853.0 Nb \]

- \( r = 0.95 \); Root Mean Square Deviation = 18 HV_{10}
\[ \Delta t_{8/5} = 1000 \text{ s} \]

\[ HV_{10} = 37.1 + 282.2C + 44.1Mn - 60.9Si + 113.2Cr + 173.4Cu + 124.7Mo + 420.0V + 236.0Ti + 550.0Nb \]

- \( r = 0.98 \); Root Mean Square Deviation = 13 HV\(_{10}\)

Observations:
- These formulas were determined using data got from samples submitted to a thermomechanical rolling simulation in a dilatometer. Thus it includes the effects of hot forming over austenite decomposition.
Austenitization: 1150°C, 5 min; 20% strain @975°C; 10 s holding; 20% strain @950°C; 10 s holding; 35% @800°C; final cooling.

Notation:
- \( \Delta t_{8/5} \): Time Between 800°C and 500°C [s]
- \( HV_{10} \): Vickers Hardness, 10 kg Load
- \textbf{Alloy Amount}: [weight %]

Source: LOTTER, U. 
\textit{Aufstellung von Regressionsgleichungen zur Beschreibung des Umwandlungsverhaltens beim thermomechanischen Walzen.} Technische Bericht, Thyssen Stahl, Duisburg, 1988, 136 p.

\[ HV = HV_{\text{max}} - (HV_{\text{max}} - HV_0) \exp \left( -\frac{0.6U \Delta t_0^n}{\Delta t^n - \Delta t_0^n} \right) \]

\[ \Delta t_0 = C \exp \left[ 2.3 \left( a_0 - \frac{0.04}{\sqrt{d}} + \sum a_i C_i \right) \right] \]
\[ n = C \exp \left[ 2.3 \left( \beta_0 + \frac{0.03}{\sqrt{d}} + \frac{0.09}{C} - \sum \beta_i C_i^{0.43} \right) \right] \]

\[ U = 1 + p \exp \left[ -0.35 \left( \ln \frac{\Delta t}{\Delta t_M} \right)^2 \right] \]

\[ \Delta t_M = \Delta t_0 \cdot 12^n \]

\[ p = \left( \frac{C}{\Delta t_0} \right)^{3.1} + \exp \left[ 2.3 \left( \gamma_0 + \sum \gamma_i C_i \right) \right] \]

\[ HV_0 = HV_{\text{min}} + 0.055 (HV_{\text{max}} - 300) \left( 9 - \log \Delta t_{700}^{300} \right) \]

\[ HV_{\text{min}} = 63 + \sum \mu_i Z_i^{0.75} \]

Notation:
- **HV\text{max}**: Maximum Hardness for a Martensitic Structure
- **C**: Carbon [weight \%]
- **C_i**: Alloy Content [weight \%]
- **d**: Mean Austenite Grain Size [mm]
- **\Delta t_{700}^{300}**: Cooling Time Between 700°C and 300°C [s]
- **HV\text{min}**: Minimum Hardness at Equilibrium Calculated According to the Substitution Solid Solution Hardening Effect Proposed by Lacy and Gensamer
- **Z_i**: Concentration of the Element \(i\) in solid solution with ferrite at equilibrium [at \%]
μᵢ: Action Coefficient of the Element i, as described in the following table:

<table>
<thead>
<tr>
<th>Element</th>
<th>Mn</th>
<th>Si</th>
<th>P</th>
<th>Ni</th>
<th>Cr</th>
<th>Mo</th>
<th>V</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>μᵢ</td>
<td>14.27</td>
<td>22.42</td>
<td>61.16</td>
<td>12.44</td>
<td>2.84</td>
<td>19.57</td>
<td>8.15</td>
<td>22.42</td>
</tr>
</tbody>
</table>

General Constants: α₀ = 0.89, β₀ = 1.22, γ₀ = 1.82

Constants: aᵢ, βᵢ and γᵢ according to the alloy element:

<table>
<thead>
<tr>
<th>Element i</th>
<th>aᵢ</th>
<th>βᵢ</th>
<th>γᵢ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mn</td>
<td>0.39</td>
<td>0.94</td>
<td>1.40</td>
</tr>
<tr>
<td>Si</td>
<td>0.20</td>
<td>0.15</td>
<td>0.80</td>
</tr>
<tr>
<td>Ni</td>
<td>0.22</td>
<td>0.40</td>
<td>0.12</td>
</tr>
<tr>
<td>Cr</td>
<td>0.67</td>
<td>0.09</td>
<td>2.40</td>
</tr>
<tr>
<td>Mo</td>
<td>0.17</td>
<td>0.72</td>
<td>0.79</td>
</tr>
<tr>
<td>V</td>
<td>0.20</td>
<td>0.50</td>
<td>0.90</td>
</tr>
<tr>
<td>Nb</td>
<td>0.40</td>
<td>1.20</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Observations:
- Equations Valid Within the Following Chemical Composition Range: 0.05% ≤ C ≤ 0.80%, 0.50% ≤ Mn ≤ 2.5%, 0.15% ≤ Si ≤ 0.35%, Ni ≤ 1.0%, Cr ≤ 1.5%, Mo ≤ 0.5%, V ≤ 0.1%, Nb ≤ 0.040%.


. Trzaska

. Specific equations:

\[ HV_{ap} = -73 + 253C + 52Mn + 10Si + 36Cr + 8Ni + 20Mo + 80V + 0.11T_A + 12.5 \sqrt{V_c} \]
\[ HV_n = 200 + 824C + 44Mn + 14Cr + 9Ni + 171V + 78.5Cu + 4.13\sqrt[3]{v_c} \]

General equation:

\[ HV = 3.7 + 225C + 82Mn + 28Si + 55Cr + 28Ni + 53.5Mo + 147V + 71Cu + 0.09T_A - 3.8\sqrt[3]{v_c} + 68C\sqrt[3]{v_c} - 42W_f - 69W_p - 32.5W_b + 72W_m \]

\[ W_x = \begin{cases} 0 & \text{if } S_x \leq N \\ 1 & \text{if } S_x > N \end{cases} \]

\[ S_x = \frac{e^{K_x}}{1 + e^{K_x}} \]

\[ K_f = 18.4 - 15.4C - 1.9Mn + 0.7Si - 2.5Cr - 1.5Ni - 4.8Mo + 2.4V + 1.4Cu - 0.004T_A + 4\sqrt[3]{v_c} \]

\[ K_p = 12 - 1.4C - 2.3Mn - 2.3Cr - 1.4Ni - 6Mo + 3.9V - 0.002T_A - 1.2\sqrt[3]{v_c} \]

\[ K_b = 1.3 - 3.7C + 0.45Mn + 0.2Cr + 0.18Ni + 1.9Mo - 0.17\sqrt[3]{v_c} + 0.57(4.35 - \sqrt[3]{v_c})^2 \]

\[ K_m = -16.5 + 4.7C + 2.6Mn + 0.6Si + 2.4Cr + 1.2Ni + 1.9Mo + 4.8Cu + 0.006T_A + 1.1\sqrt[3]{v_c} \]

Notation:

\(HV_{ap}\): Ferrite-Pearlite Hardness [Vickers]
\(HV_m\): Martensite Hardness [Vickers]
HV: General Hardness [Vickers]
Alloy Content: [weight %]
TA: Austenitizing Temperature [°C]
v_c: Cooling Rate [°C/min]
x: f (ferrite), p (pearlite), b (bainite) or m (martensite)
N: 0.5 for ferrite (f), pearlite (p) and martensite (m); 0.4 for bainite (b).

Observations:
- Formula valid within the following range: 0.06% ≤ C ≤ 0.68%, 0.13% ≤ Mn ≤ 2.04%, 0.12% ≤ Si ≤ 1.75%, Cr ≤ 2.30%, Ni ≤ 3.85%, Mo ≤ 1.05%, V ≤ 0.38% and Cu ≤ 0.38%.
- Additional conditions: Mn+Cr ≤ 3.6%. Mn+Cr+Ni ≤ 5.6%, Cr+Ni ≤ 5.3%, Mn+Ni ≤ 4.5%.
- HV: r² = 0.847, standard error = 62.3 HV, mean absolute error = 48.5 HV.
- HV_{ap}: r² = 0.743, standard error = 24.4 HV, mean absolute error = 19.4 HV.
- HV_m: r² = 0.855, standard error = 39.9 HV, mean absolute error = 30.5 HV.

- **Hardness After Tempering**

  * Spies

\[
HB = 2.84 \times HRC + 75 C - 0.78 Si + 14.24 Mn + 14.77 Cr + 128.22 Mo - 54.0 V - 0.55 T + 435.66
\]

Notation:
- **HB**: Brinell Hardness After Hardening and Tempering
- **HRC**: Rockwell Hardness (C Scale) After Hardening
- **Alloy Content**: [weight %]
- **T**: Tempering Temperature [°C]

Observations:
- This equation is valid within the following ranges: **HRC**: 20~65; **C**: 0.20~0.54%; **Mn**: 0.50~1.90%; **Si**: 0.17~1.40%; **Cr**: 0.03~1.20%; **T**: 500~650°C.

- **Hardness After Welding**

  . **Dearden & O’Neill**

  \[ HV_{\text{max}} = 1200C_{\text{EQ,Dearden}} - 200 \]

  \[ C_{\text{EQ,Dearden}} = C + \frac{Mn}{6} + \frac{Mo}{4} + \frac{Cr+V}{5} + \frac{Cu}{13} + \frac{Ni}{15} + \frac{P}{2} \]

  Notation:
  - \( C_{\text{EQ,Dearden}} \): Equivalent Carbon (Dearden) [%]
  - **Alloy Content**: [weight %]
  - \(HV_{\text{max}}\) = Maximum Hardness [Vickers]

  Observations:
  - This equation calculates maximum hardness after welding.


  . **Khan**

  \[ HV = 188 + 630C_{\text{EQ,Yurioka}} \]

  Notation:
  - \( C_{\text{EQ,Yurioka}} \): Equivalent Carbon for Pipeline Steels [%]

  \[ C_{\text{EQ,Yurioka}} = C + A(C)\left(\frac{Mn}{6} + \frac{Si}{24} + \frac{Cr + Mo + V}{5} + \frac{Cu}{15} + \frac{Ni}{20} + \frac{Nb}{5} + 5B\right) \]
\[ A(C) = 0.75 + 0.25 \tanh \left[ 20 (C - 0.12) \right] \]

**Alloy Content**: [weight %]

Observations:
- \( HV \) is the fusion zone hardness for a single pulse resistance spot weld with a 5 cycle hold time, calculated with \( r = 0.961 \).


**Shinozaki**

\[ HV = 78 + 331C_{\text{EQ_FBW}} \]

\[ C_{\text{EQ_FBW}} = C + \frac{Mn}{5} + \frac{Si}{15} + \frac{Cr}{9} + 7 \nb (1 - 10C) + \frac{V (50 C - 1)}{3} + 1.3 Ti (1 - 5 C) + \frac{Mo (1 - 6 C)}{2} + 29 B (11 C - 1) \]

Notation:
- \( C_{\text{EQ_FBW}} \): Equivalent Carbon Designed Specifically for Flash Butt Welding [%]
- **Alloy Content**: [weight %]
- **HV**: Hardness at the Welding Interface [Vickers]

- **Hardness-Tensile Properties Equivalence**

**Hashemi**

\[ YS = 2 \ HV + 105 \]

\[ UTS = 1.3 \ HV + 344 \]

Notation:
- **YS**: Yield Strength [MPa]
- **UTS**: Ultimate Tensile Strength [MPa]
- **HV**: Hardness [Vickers]

Observations:
- Equations determined from an API X65 gas pipe steel with 0.071% C, 1.515% Mn, 0.209% Si, 0.044% Nb, 0.017% Ti, 0.042% V and 0.157% Cr.


**Hajibagheri**

\[ YS = 0.5915 \ HV_{10} + 406.79 \]

\[ UTS = 0.6788 \ HV_{10} + 486.44 \]

Notation:
- **YS**: Yield Strength [MPa]
- **UTS**: Ultimate Tensile Strength [MPa]
- **HV_{10}**: Hardness [Vickers, 10 kg Load]

Observations:
- Equations determined from an API X70 gas pipe steel with 0.050-0.100% C, 1.45-1.60% Mn, 0.17-0.37% Si, 0.030-0.040% Nb, 0.020-0.050% Ti, 0.007-0.050% V, 0.04-0.20% Cr and 0.001-0.008% Mo.

- **Hot Strength of Steel**

  . Alloy Elements Effect

![Graph showing the effect of alloy elements on hot strength increase factor vs atomic diameter ratio](image-url)
### Alloy Element

<table>
<thead>
<tr>
<th>Alloy Element</th>
<th>$\frac{\Delta \sigma_{0.3}}{\sigma_{0.3}}$₀.₃ [%/% atomic]</th>
<th>$\frac{\Delta \sigma_{0.3}}{\sigma_{0.3}}$₀.₃ [%/% weight]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mn</td>
<td>0.9838</td>
<td>2.13</td>
</tr>
<tr>
<td>Si</td>
<td>0.5054</td>
<td>7.52</td>
</tr>
<tr>
<td>Cr</td>
<td>0.9317</td>
<td>1.93</td>
</tr>
<tr>
<td>Mo</td>
<td>1.70061</td>
<td>12.31</td>
</tr>
<tr>
<td>Cu</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Ni</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Nb</td>
<td>1.6527</td>
<td>127.06</td>
</tr>
<tr>
<td>V</td>
<td>0.9130</td>
<td>31.76</td>
</tr>
<tr>
<td>Ti</td>
<td>0.8584</td>
<td>59.76</td>
</tr>
</tbody>
</table>

**Observations:**

- Plot generated from data available in the original source.


### Misaka

$$\tilde{\sigma} = \exp \left[ 0.126 - 1.75 C + 0.594 C^2 + \frac{2851 + 2968 C - 1120 C^2}{T} \right] \varepsilon^{0.21} \left( \frac{d \varepsilon}{dt} \right)^{0.13}$$

**Notation:**

...
\[ \sigma = f \ g \ \exp \left[ 0.126 - 1.75C + 0.594C^2 + \frac{(2851 + 2968C - 1120C^2)}{T} \right] \varepsilon^{0.21} \left( \frac{d\varepsilon}{dt} \right)^{0.13} \]

\[ f = 0.916 + 0.18Mn + 0.389V + 0.191Mo + 0.004 Ni \]

If \( T \), expressed in Celsius degrees, is between \( Ar_3 \) and \( Ar_1 \), then \( g \) must be calculated according to the formula below. Otherwise \( g \) is equal to unity.

\[ g = 0.7893 + 0.769C \]

\[ Ar_3 = 974.76 - 734.65C \]
\[ \text{Ar}_i = 876.81 - 336.26 \ C \]

Notation:
- \( \sigma \): Steel Mean Flow Stress \([\text{kgf/mm}^2]\)
- \( f \): Effect of alloy elements on Mean Flow Stress.
- \( g \): Softening Factor Due to Intercritical Deformation
- \( C \): Carbon Content [weight %]
- \( T \): Absolute Temperature [K]
- \( \varepsilon \): True Strain
- \( t \): Time [s]
- \( \text{Mn} \): Manganese Content [weight %]
- \( \text{V} \): Vanadium Content [weight %]
- \( \text{Mo} \): Molybdenum Content [weight %]
- \( \text{Ni} \): Nickel Content [weight %]
- \( \text{Ar}_3 \): Temperature of Start Austenite Transformation in Proeutectoid Ferrite \( [\text{°C}] \)
- \( \text{Ar}_1 \): Temperature of Finish Austenite Transformation in Proeutectoid Ferrite \( [\text{°C}] \)

Observations:
- The mean flow stress calculated by this equation is given in effective (von Mises) units, as it was determined under plane strain conditions.


\[ \overline{\sigma} = a \sqrt{\rho} \]
\[
\rho = \rho_n (1 - X_{dyn}) + \rho_s X_{dyn}
\]

\[
\rho_n = \frac{c \left(1 - e^{-b\varepsilon}\right)}{b} + \rho_0 \ e^{-b\varepsilon}
\]

\[
b = 9850 \varepsilon^{-0.315} \left(\frac{-8000}{T}\right)
\]
(Senuma 1984)

\[
b = 6227 \varepsilon^{-0.28} \left(\frac{-7500}{T}\right)
\]
(Wang & Tseng)

\[
c = 1.0 \times 10^1
\]
(Senuma 1984)

\[
c = 8.5 \times 10^{10} \left(1 + \frac{1}{\sqrt{D_0}}\right)
\]
(Yada & Senuma)

\[
\varepsilon_c = 4.76 \times 10^{-4} \left(\frac{8000}{T}\right)
\]
(Senuma)

\[
\varepsilon_c = 0.05 \left(\frac{2500}{T}\right)
\]
(Wang & Tseng 1996)

If \(\varepsilon \geq \varepsilon_c\):

\[
X_{dyn} = 1 - e^{-0.693 \left(\frac{\varepsilon - \varepsilon_c}{\varepsilon_0.5}\right)^2}
\]
\[ \rho_s = 3.8242 \times 10^9 \varepsilon^{0.2} \left[ \frac{(1613 - T)}{290} \right]^2 \]

. According to Senuma (1984):

\[ \varepsilon_{0.5} = 1.144 \times 10^{-5} D_0^{0.28} \varepsilon^{0.05} e^{\left( \frac{6420}{T} \right)} \]

. According to Wang & Tseng:

\[ \varepsilon_{0.5} = 1.07 \times 10^{-2} D_0^{0.28} \varepsilon^{0.03} e^{\left( \frac{2650}{T} \right)} \]

\[ D_{dyn} = 22600 \varepsilon^{-0.27} e^{\left( \frac{-8670}{T} \right)} \]

If \( X_{dyn} > 0.95 \):

. According to Yada & Senuma:

\[ D_p = D_{dyn} + (D_{pd} - D_{dyn}) \left[ 1 - e^{-295 \varepsilon^{51} e^{\left( \frac{2000}{T} \right)}} \right] \]
According to Wang & Tseng:

\[ D_p = D_{dy} + 1.1 (D_{pl} - D_{dy}) \left[ 1 - e^{-295 \varepsilon_{pl} \left( \frac{t}{18000} \right)} \right] \]

\[ D_{pl} = 5380 e^{-\left( \frac{-6840}{T} \right)} \]

If \( \varepsilon < \varepsilon_c \):

\[ D_{st} = \frac{5}{(\varepsilon S_v)^{0.6}} \]

\[ S_v = \frac{24 (0.4914 e^{\varepsilon} + 0.155 e^{-\varepsilon} + 0.1433 e^{-3\varepsilon})}{\pi D_0} \]

\[ X_{st} = 1 - e^{-0.693 \left[ \frac{(u - t_s)}{t_{0.5}} \right]^2} \]

According to Senuma (1984):

\[ t_{0.5} = \frac{0.286 \times 10^{-7}}{\sqrt{S_v}} e^{-0.2} \varepsilon^{-2} e^{\left( \frac{18000}{T} \right)} \]
. According to Wang & Tseng:

\[
t_{0.5} = \frac{2.2 \times 10^{-12}}{\sqrt{S_v}} \varepsilon^{-0.2} \varepsilon^{-2} e^{\left(\frac{30000}{T}\right)}
\]

\[
t_{0.95} = 4.322 t_{0.5}
\]

If \(X_{st} < 0.95\) (i.e., \(t_{ip} < t_{0.95}\)):

\[
D_a = D_{st} (0.2 + 0.8 X_{st})
\]

If \(X_{st} \geq 0.95\) and \(t_{ip} \geq t_{0.95}\):

. According to Senuma (1984):

\[
D_s = \sqrt{D_a^2 + 1.44 \times 10^{12} t_s e^{\left(\frac{63800}{T}\right)}}
\]

. According to Wang & Tseng:

\[
D_s = \sqrt{D_a^2 + 1.44 \times 10^{12} t_s e^{\left(\frac{32100}{T}\right)}}
\]
\[ t_g = t_p - t_{0.95} \]

In case of multipass hot rolling:

\[
\rho_R = \rho_i (1 - X_{\text{dyn}})(1 - X_d) + \rho_i X_{\text{dyn}} \left( \frac{D_{\text{pd}} - D_p}{D_{\text{pd}} - D_{\text{dyn}}} \right)
\]

\[
\rho_i = \rho_n e^{\left( \frac{-90.5}{T} \right)}
\]

\[
\ln \left( \frac{\rho_i b - c}{\rho_n b - c} \right) = \varepsilon_r
\]

Notation:
- \( \sigma \): Steel Mean Flow Stress [kgf/mm²]
- \( \rho \): Dislocation Density [cm⁻²]
- \( \rho_0 \): Initial Dislocation Density [cm⁻²]
- \( \rho_n \): Dislocation Density in the Dynamically Recovered Region [cm⁻²]
- \( \rho_s \): Dislocation Density in the Dynamically Recrystallized Region [cm⁻²]
- \( \rho_i \): Dislocation Density After Deformation/Static Recovery [cm⁻²]
- \( \rho_t \): Remaining Dislocation Density in the Dynamically Recovered Region [cm⁻²]
- \( X_{\text{dyn}} \): Fraction of Dynamic Recrystallization
- \( X_{\text{st}} \): Fraction of Static Recrystallization
- \( T \): Absolute Temperature [K]
- \( \varepsilon \): True Strain
- \( \varepsilon_c \): Critical Strain for the Onset of Dynamic Recrystallization
- \( \varepsilon_{0.5} \): Strain Required for 50% Dynamic Recrystallization
- \( \varepsilon_r \): Residual Strain After One Pass of Hot Rolling
\( \dot{\varepsilon} \): Strain Rate [s\(^{-1}\)]

**D\(_0\)**: Grain Size Before Deformation [\( \mu m \)]

**D\(_{\text{dy}}\)**: Dynamically Recrystallized Grain Size [\( \mu m \)]

**D\(_p\)**: Transition Grain Size from **D\(_{\text{dy}}\)** to **D\(_{\text{pd}}\)** at a time **t** after deformation [\( \mu m \)]

**D\(_{\text{pd}}\)**: Grain Size resulted from driving force due to the decrease of dislocation density [\( \mu m \)]

**D\(_{\text{st}}\)**: Statically Recrystallized Grain Size [\( \mu m \)]

**D\(_u\)**: Mixed Grain Size Due to Incomplete Static Recrystallization [\( \mu m \)]

**S\(_v\)**: Nucleation Site Area [\( \mu m^{-1} \)]

**t**: Time [s]

**t\(_s\)**: Incubation Time for Static Recrystallization [s]

**t\(_{0.5}\)**: Time Required for 50% Static Recrystallization [s]

**t\(_{0.95}\)**: Time Required for 95% Static Recrystallization [s]

**t\(_{ip}\)**: Time Interval Between Successive Rolling Passes [s]

**t\(_g\)**: Time Available for Grain Growth [s]

**t\(_a\)**: Time After Deformation [s]

**Observations:**

- This model is very interesting as it links hot strength with microstructural evolution.
- The mean flow stress calculated by this equation is given in effective (von Mises) units, as it was determined under plane strain conditions.
- The value of constant **a** depends on steel composition. For instance:
  - 0.00175 MN/m\(^2\) (Senuma 1984: 0.08-0.81% C, 0.62-1.14% Mn, 0.20-0.24% Si)
  - 0.00165 MN/m\(^2\) (Yada & Senuma: 0.05-0.40% C, 0.00-1.00% Mn, 0.00-0.50% Si)
  - 0.00180 MN/m\(^2\) (Wang & Tseng: 0.05-0.81% C, 0.20-1.50% Mn, 0.01-0.50% Si)
- Suggested value for **p\(_0\)**: 1 x 10\(^{-8}\) cm\(^{-2}\) (Wang & Tseng)
- **t\(_s\)** can be assumed as being zero as it is negligibly short for the deformation conditions of hot flat rolling (Wang & Tseng).
- If \( \varepsilon_r \) from the former pass is greater than 0, then it must be added to value of \( \varepsilon \) of the next pass.

**Sources:**


. Shida

Calculation algorithm expressed in Visual Basic:

```vbnet
Function Shida(C, T, Def, VelDef)

Dim nShida, Td, g, Tx, mShida, SigF As Single

nShida = 0.41 - 0.07 * C
Td = 0.95 * (C + 0.41) / (C + 0.32)
T = (T + 273) / 1000
If T >= Td Then
    g = 1
    Tx = T
    mShida = (-0.019 * C + 0.126) * T + (0.075 * C - 0.05)
Else
End If

```


\[ g = 30 \times (C + 0.9) \times (T - 0.95 \times (C + 0.49) / (C + 0.42)) ^ 2 + (C + 0.06) / (C + 0.09) \]
\[ Tx = Td \]
\[ mShida = (0.081 \times C - 0.154) \times T + (-0.019 \times C + 0.207) + 0.027 / (C + 0.32) \]
\[ \text{End If} \]
\[ \text{SigF} = 0.28 \times g \times \exp(5 / Tx - 0.01 / (C + 0.05)) \]
\[ \text{Shida} = 2 / \sqrt{3} \times \text{SigF} \times (1.3 \times (\text{Def} / 0.2) ^ nShida - 0.3 \times (\text{Def} / 0.2)) \times (\text{VelDef} / 10) ^ mShida \]

**End Function**

**Notation:**
- **Shida**: Steel Mean Flow Stress [kgf/mm²]
- **C**: C content [weight %]
- **T**: Temperature [°C]
- **Def**: True Strain
- **VelDef**: Strain Rate [s⁻¹]

**Observation:**
- The mean flow stress calculated is this algorithm is already expressed in effective (von Mises) units, that is, corrected for plane strain conditions, as it is multiplied by \( 2/\sqrt{3} \).
- Equation valid for the following parameter range: \( C \leq 1.20\% ; 700 \leq \epsilon \leq 1200 ^\circ C ; \epsilon \leq 0.7 ; 0.1 \leq \dot{\epsilon} \leq 100 \ \text{s}^{-1} \).
- The effect of some alloy elements over hot strength can be considered by Shida equation. In this case carbon content must be replaced by an equivalent carbon \((C_{eq})\) content, which formula is described below:

\[
C_{eq} = C + \frac{Mn}{6} + \frac{Cr + V + Nb}{12}
\]

where \( Mn \) is the manganese content, \( Cr \) is the chromium content, \( V \) is the vanadium content and \( Nb \) is the niobium content, all expressed as weight percent.

**Sources:**

- **Jominy Curves**

  . Just

- **d < 6.4 mm**

  \[ J_d = 60 \sqrt{C} + 20 \]

- **6.4 ≤ d < 39.7 mm**

  \[ J_d = 98 \sqrt{C} + 0.00992 d^2 \sqrt{C} + 20 Cr + 6.4 Ni + 19 Mn + 34 Mo + 28 V - 19.05 \sqrt{d} + 1.80 d + 7 \]

- **C < 0.28% and 6.4 mm ≤ d < 39.7 mm**

  \[ J_d = 87 C + 14 Cr + 5.3 Ni + 16 Mn + 29 Mo - 16.8 \sqrt{d} + 1.39 d + 22 \]

- **C > 0.29% and 6.4 mm ≤ d < 39.7 mm**

  \[ J_d = 78 C + 22 Cr + 6.9 Ni + 21 Mn + 33 Mo - 16.1 \sqrt{d} + 1.17 d + 18 \]

Observation:

- Equations valid for the following chemical composition range: 0.10% ≤ C ≤ 0.60%, 0.45% ≤ Mn ≤ 1.75%, 0.15% ≤ Si ≤ 1.95%, Ni ≤ 5.0%, Cr ≤ 1.55%, Mo ≤ 0.52% and V ≤ 0.2%.

- **Equation Considering the Effect of Austenite Grain Size**
\[ J_d = 88 \sqrt{C} - 0.00553 \sqrt{C} + 19 Cr + 6.3 Ni + 16 Mn + 5 Si - 0.82 G_{\gamma} - 15.9 \sqrt{d} + 1.33 d - 2 \]

Observation:
- Equation valid for the following conditions: \(0.08\% \leq C \leq 0.56\%, \ 0.20\% \leq Mn \leq 1.88, \ Si \leq 3.80, \ Ni \leq 8.94\%, \ Cr \leq 1.97, \ Mo \leq 0.53\) and \(1.5 \leq G_{\gamma} \leq 11\).

Notation:
- \(J_d\): Hardness [Rockwell C]
- **Alloy Content**: [weight %]
- \(d\): Distance from the Cooled End [mm]
- \(G_{\gamma}\): Austenite Grain Size Index [mm]

- **Lattice Parameters of Phases**

  . **Ferrite**

  \[ a_\alpha = 2.8863[1 + 17.5 \times 10^{-6} (T - 800)] \]

  Notation:
  
  \( a_\alpha \): Ferrite Lattice Parameter [Å]
  
  \( T \): Temperature [K]

  Observations:
  
  - \( 800 \text{ K} < T < 1200 \text{ K} \)

  . **Austenite**

  \[ a_\gamma = 3.573 + 0.033C + 0.00095Mn - 0.0002Ni + 0.0006Cr + 0.0031Mo + 0.0018V \]

  Notation:
  
  \( a_\gamma \): Austenite Lattice Parameter [Å]
  
  \( T \): Temperature [K]
  
  \( \xi \): C [Atomic Fraction]

  Observations:
  
  - \( 1000 \text{ K} < T < 1250 \text{ K} \)
  
  - \( 0.0005 < \xi < 0.0365 \)

  \[ a_\gamma = (3.6306 + 0.78 \xi)[1 + (24.9 - 50 \xi) \times 10^{-6} (T - 1000)] \]
Notation:

\( a_\gamma \): Austenite Lattice Parameter [Å]

**Alloy Content**: [Weight Percent]

Observations:
- \( 1000 \, \text{K} < T < 1250 \, \text{K} \)

**Cementite**

\[
a_\gamma = 4.5234 \left[ 1 + (5.311 \times 10^{-6} - 1.942 \times 10^{-9} \, T + 9.655 \times 10^{-12} \, T^2) \right] (T - 293) \]

\[
b_\gamma = 5.0883 \left[ 1 + (5.311 \times 10^{-6} - 1.942 \times 10^{-9} \, T + 9.655 \times 10^{-12} \, T^2) \right] (T - 293) \]

\[
c_\gamma = 6.7426 \left[ 1 + (5.311 \times 10^{-6} - 1.942 \times 10^{-9} \, T + 9.655 \times 10^{-12} \, T^2) \right] (T - 293) \]

Notation:

\( a_\theta, b_\theta, c_\theta \): Cementite Lattice Parameter [Å]

**T**: Temperature [K]

Observations:
- \( 300 \, \text{K} < T < 1000 \, \text{K} \)

- **Liquid Steel Solubility Products**

  . General

  \[
  \log \left( \frac{(a_A)^m}{(a_B)^n} \right) = - \frac{A}{T} + B
  \]

  Notation:
  
  \( A_m B_n \): Precipitate Considered for Calculation
  
  \( a_x \): Alloy Content [weight %]
  
  \( T \): Temperature [K]
  
  \( A, B \): Constants of the Solubility Product, given in the table below:

<table>
<thead>
<tr>
<th>Precipitate</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>MnS</td>
<td>8236</td>
<td>5.03</td>
</tr>
<tr>
<td>TiN</td>
<td>16586</td>
<td>5.90</td>
</tr>
<tr>
<td>TiS</td>
<td>8000</td>
<td>4.00</td>
</tr>
<tr>
<td>ZrN</td>
<td>17000</td>
<td>6.38</td>
</tr>
</tbody>
</table>

  Observations:
  
  - \( a_{A_mB_n} \) is equal to one if the precipitate is pure.
  
  - \( a_{A_mB_n} \leq 1 \) if there is co-precipitation with another element.

Source: Values compiled by Rajindra Clement Ratnapuli from assorted references.
- **Liquidus Temperature of Steels**

\[ T_{\text{Liq}} = 1536 - 78\, C + 7.6\, Si + 4.9\, Mn + 34\, P + 30\, S + 5\, Cu + 3.1\, Ni + 1.3\, Cr + 3.6\, Al + 2\, Mo + 2\, V + 18\, Ti \]

Notation:
- \( T_{\text{Liq}} \): Steel Melting Temperature [°C]
- **Alloy Content**: [weight %]

Niobium Carbide Precipitation During Rolling

- **Poisson Ratio**

  . **Definition**

  \( \nu \): Poisson Ratio

  . Elastic Range: 0.3
  . Plastic Range: 0.5


  . **Fletcher**

<table>
<thead>
<tr>
<th>Temperature [^{°C}]</th>
<th>( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>0.327</td>
</tr>
<tr>
<td>700</td>
<td>0.335</td>
</tr>
<tr>
<td>800</td>
<td>0.344</td>
</tr>
<tr>
<td>900</td>
<td>0.352</td>
</tr>
<tr>
<td>1000</td>
<td>0.360</td>
</tr>
</tbody>
</table>

- **Precipitate Isothermal Solubilization Kinetics**

\[ t = \frac{r_0^2}{2 c D} \]

Notation:
- \( A_m B_n \): Spheric precipitate considered for calculation
- \( t \): Time for solubilization of the precipitate [s]
- \( r_0 \): Radius of the precipitate [m, cm, or mm]

\[ c = \frac{C_i - C_m}{C_p - C_i} \approx \frac{C_i}{C_p} \]

- \( C_m \): Solute concentration in the bulk metal [%]
- \( C_i \): Solute concentration in the precipitate/matrix interface [%]

\[ C_i = 10^{\left( \frac{-A + B}{T} \right)} \]

- \( T \): Temperature [K]
- \( A, B \): Constants of the Solubility Product, given in the table at the topic *Austenite Solubilization Products.*
- \( a_B \): Alloy content [weight percent]
- \( C_p \): Solute content in the precipitate [%]

\[ C_p = \frac{m M_A}{m M_A + n M_B} \]

- \( M_x \): Atomic mass of the element [g]
### Solubility and Diffusion Coefficients

**C<sub>m</sub>:** Solute content in a position far away from the precipitate [%]

**D:** Solute Diffusion Coefficient [m<sup>2</sup>/s, cm<sup>2</sup>/s or mm<sup>2</sup>/s], calculated according to the general equation below:

\[
D = D_0 \exp \left( -\frac{Q}{RT} \right)
\]

- **D<sub>0</sub>:** Constant
- **Q:** Activation Energy for Diffusion [J] or [cal]
- **R:** Universal Gas Constant, 1.981 cal/mol.K
- **R':** Universal Gas Constant, 8.314 J/mol.K

### Table of Diffusion Coefficients

<table>
<thead>
<tr>
<th>Element</th>
<th>Phase</th>
<th>Equation</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>Ferrite</td>
<td>( D ) [m&lt;sup&gt;2&lt;/sup&gt;/s] = 0.30 \times 10^{-2} \times \exp(-234500/R'T)</td>
<td>Pickering</td>
</tr>
<tr>
<td></td>
<td>Austenite</td>
<td>( D ) [m&lt;sup&gt;2&lt;/sup&gt;/s] = 0.49 \times 10^{-4} \times \exp(-284100/R’T)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Austenite</td>
<td>( D ) [m&lt;sup&gt;2&lt;/sup&gt;/s] = 2.10 \times 10^{-3} \times \exp(-286000/R’T)</td>
<td>Borggren</td>
</tr>
<tr>
<td>B</td>
<td>Austenite</td>
<td>( D ) [m&lt;sup&gt;2&lt;/sup&gt;/s] = 2 \times 10^{-4} \times \exp(-87864/R’T)</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Ferrite</td>
<td>( D ) [cm&lt;sup&gt;2&lt;/sup&gt;/s] = 0.02 \times \exp(-20100/RT)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ferrite</td>
<td>( D ) [m&lt;sup&gt;2&lt;/sup&gt;/s] = 0.62 \times 10^{-6} \times \exp(-80400/R’T)</td>
<td>Pickering</td>
</tr>
<tr>
<td></td>
<td>Austenite</td>
<td>( D ) [m&lt;sup&gt;2&lt;/sup&gt;/s] = 0.10 \times 10^{-4} \times \exp(-135700/R’T)</td>
<td></td>
</tr>
<tr>
<td>Cr</td>
<td>Ferrite</td>
<td>( D ) [cm&lt;sup&gt;2&lt;/sup&gt;/s] = 8.52 \times \exp(-59900/RT)</td>
<td></td>
</tr>
<tr>
<td>Element</td>
<td>Phase</td>
<td>Equation</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
<td>----------</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td>Austenite</td>
<td>( D = 10.80 \times \exp(-69700/RT) )</td>
<td></td>
</tr>
<tr>
<td>Mn</td>
<td>Austenite</td>
<td>( D = 0.49 \times 10^{-4} \times \exp(-284100/RT) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Austenite</td>
<td>( D = 7.00 \times 10^{-5} \times \exp(-286000/RT) )</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>Austenite</td>
<td>( D = 0.91 \times 10^{-4} \times \exp(-168600/RT) )</td>
<td></td>
</tr>
<tr>
<td>Nb</td>
<td>Austenite</td>
<td>( D = 5.30 \times 10^{-2} \times \exp(-344600/RT) )</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>Austenite</td>
<td>( D = 2.90 \times \exp(-55000/RT) )</td>
<td></td>
</tr>
<tr>
<td>Element</td>
<td>Phase</td>
<td>(D \text{[m}^2\text{/s}]=) (A \cdot e^{\frac{-E}{RT}})</td>
<td>Source</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
<td>------------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>Si</td>
<td>Austenite</td>
<td>(7.00 \times 10^{-4} e^{\frac{-286000}{RT}})</td>
<td>Borggren</td>
</tr>
<tr>
<td>Ti</td>
<td>Austenite</td>
<td>(1.50 \times 10^{-5} e^{\frac{-251000}{RT}})</td>
<td>Borggren</td>
</tr>
<tr>
<td>V</td>
<td>Ferrite</td>
<td>(3.92 e^{\frac{-57600}{RT}})</td>
<td>Pickering</td>
</tr>
<tr>
<td></td>
<td>Ferrite</td>
<td>(0.61 \times 10^{-4} e^{\frac{-267100}{RT}})</td>
<td>Pickering</td>
</tr>
<tr>
<td></td>
<td>Austenite</td>
<td>(0.25 e^{\frac{-63100}{RT}})</td>
<td>Pickering</td>
</tr>
<tr>
<td></td>
<td>Austenite</td>
<td>(0.25 \times 10^{-4} e^{\frac{-264200}{RT}})</td>
<td>Pickering</td>
</tr>
</tbody>
</table>

Sources:


- Information compiled by Rajindra Clement Ratnapuli from assorted references.
- **Relationships Between Chemical Composition x Process x Microstructure x Properties**

  . **Acicular Ferrite/Low Carbon Bainite Steels**

  \[
  YS = 88 + 37 \, Mn + 83 \, Si + 2900 \, N_{\text{sol}} + \frac{15.1}{d_L} + \sigma_{\text{disc}} + \sigma_{\text{ppt}}
  \]

  \[
  TS = 246 + 1900 \, C + 230 \, (Mn + Cr) + 185 \, Mo + 90 \, W + 125 \, Ni + 65 \, Cu + 385 \, (V + Ti)
  \]

  \[
  ITT = -19 + 44 \, Si + 700 \, \sqrt{N_{\text{sol}}} + 0.26 (\sigma_{\text{disc}} + \sigma_{\text{ppt}}) - \frac{11.5}{d}
  \]

  **Notation:**
  - **YS**: Yield Strength at 0.2% Real Strain [MPa]
  - **TS**: Tensile Strength [MPa]
  - **ITT**: Impact Transition Temperature for 50% Tough Fracture [°C]
  - **Alloy Content**: [weight %]
  - **N_{\text{sol}}**: Solubilized (Free) Nitrogen [%]
  - **d_L**: BainiteFerrite Lath Size [mm]
  - **\sigma_{\text{disc}}**: Strength Due to Dislocations [MPa]
  - **\sigma_{\text{ppt}}**: Precipitation Strengthening According to the Ashby-Orowan Model [MPa]
  - **N_{\text{sol}}**: Solubilized (Free) Nitrogen [%]
  - **d**: Mean Spacing between High Angle Boundaries (“Packet” or Prior Austenite Grain Boundaries)

  \[
  \Delta\sigma_{\text{disc}} = \alpha \, \mu \, b \, \sqrt{\rho} = 1.2 \times 10^{-3} \, \sqrt{\rho} \, (\text{PICKERING}) \, \text{or} \, 8 \times 10^{-4} \, \sqrt{\rho} \, (\text{KEH})
  \]

  \[
  \Delta\sigma_{\text{ppt}} = \frac{5.9 \, \sqrt{f}}{\bar{x}} \, \ln \left( \frac{\bar{x}}{2.5 \times 10^{-4}} \right)
  \]
Notation:

- $\alpha$: Empirical Constant
- $\mu$: Shear Modulus [MPa]
- $b$: Burger’s Vector [cm]
- $\rho$: Dislocation Density [lines/cm²]
- $f$: Volume Fraction of the Precipitate
- $x$: Mean Planar Intercept Diameter of the Precipitate [$\mu$m]

Sources:


**. C-Mn Mild Steels (Pickering)**

\[
YS = 53.9 + 32.3 \text{Mn} + 83.2 \text{Si} + 354.2 \sqrt{N_{\text{sol}}} + \frac{17.4}{\sqrt{d}}
\]

\[
TS = 294.1 + 27.7 \text{Mn} + 83.2 \text{Si} + 2.85 \text{Pearl} + \frac{7.7}{\sqrt{d}}
\]

\[
\frac{d\sigma}{d\varepsilon} = 370 + 120 \text{C} + 23.1 \text{Mn} + 116 \text{Si} + 554 \text{P} + 143 \text{Sn} + 1509 N_{\text{sol}} + \frac{15.4}{\sqrt{d}}
\]

\[
\varepsilon_{\text{unif}} = 0.28 - 0.20 \text{C} - 0.25 \text{Mn} - 0.044 \text{Si} - 0.039 \text{Sn} - 1.2 N_{\text{sol}}
\]
\[ \varepsilon_{\text{tot}} = 1.40 - 2.90 \, C + 0.20 \, Mn + 0.16 \, Si - 2.2 \, S - 3.9 \, P + 0.25 \, Sn + \frac{0.017}{\sqrt{d}} \]

\[ 50\% \, ITT = -19 + 44 \, Si + 700 \sqrt{N_{\text{sol}}} + 2.2 \, Pearl - \frac{11.5}{\sqrt{d}} \]

\[ \Delta Y = 12.32 - 19250 \, N_{\text{sol}} + 162 \, Mn + 462 \, O \]

Notation:
- **YS**: Yield Strength at 0.2% Real Strain [MPa]
- **TS**: Tensile Strength [MPa]
- \(d\sigma/d\varepsilon\): Strain Hardening Coefficient at 0.2% Real Strain [1/MPa]
- \(\varepsilon_{\text{unif}}\): Uniform Elongation, Expressed as Real (Logarithmic) Strain
- \(\varepsilon_{\text{tot}}\): Total Elongation, Expressed as Real (Logarithmic) Strain
- **Pearl**: Pearlite Fraction in Microstructure [%]
- **50% ITT**: Impact Transition Temperature for 50% Tough Fracture [°C]
- **\(\Delta Y\)**: Strain Ageing After 10 Days at Room Temperature [MPa]
- **Alloy Content**: [weight %]
- **d**: Grain Size [mm]

\[ TS = 237 + 29 \, Mn + 79 \, Si + 700 \, P + \frac{7.24 \, F}{\sqrt{d}} + 500 \,(1 - F) \]

Notation:
- **YS**: Yield Strength at 0.2% Real Strain [MPa]
- **TS**: Tensile Strength [MPa]
- **Alloy Content**: [weight %]
- **F**: Ferrite Fraction
- **d**: Ferritic Grain Size [mm]


**C-Mn Steels Processed at a Hot Strip Mill**

\[ d = 11.5 - 2.2 \,(6 \, C + Mn + 30 \, P + 35 \, S + 23 \, Al + 0.01 \,(723 - T_{\text{coil}}) + 0.01 \, e_{\text{sol}} - 0.002 \, T_{\text{fin}} - 100 \, N_{\text{sol}}) \]

\[
Pearl = \alpha \frac{C_{\text{eq}} - 0.06}{0.78} \times 100
\]

\[ S_0 = \frac{0.1}{723 - T_{\text{coil}}} \]

\[ YS = 99.08 \,(38.2 + \frac{0.016 \, Pearl}{\sqrt{S_0}} + 5.5 \, Mn + 43 \, Si + 114 \, P - 45 \, S + 31 \sqrt{N_{\text{sol}}} + \frac{12.6}{\sqrt{d}} - 0.02 \, T_{\text{fin}}) \]
\[ TS = 130.47 \left( 19.8 + \frac{0.004 \text{Pearl}}{\sqrt{S_0}} \right) + 8.03 \text{Mn} + 41.4 \text{Si} + 57.7 \text{P} - 69 \text{S} + 262 \sqrt{\text{N}_{\text{sol}}} + \frac{11.5}{\sqrt{d}} \]

\[ \varepsilon = 100 \left( 0.000096 \text{Pearl} S_0 - 0.05 \text{Mn} - 4.23 \text{P} - 4.36 \text{S} + 2.37 \text{Sn} - 1.16 \sqrt{\text{N}_{\text{sol}}} + \frac{0.12}{\sqrt{d}} + 0.0006 T_{\text{fin}} \right) \]

Notation:
- \text{YS}: Yield Strength at 0.2\% Real Strain [MPa]
- \text{TS}: Tensile Strength [MPa]
- \varepsilon: Total Elongation [%]
- \text{d}: Ferrite Grain Size [\mu m]
- \text{Alloy Content}: [weight %]
- \text{T}_{\text{coil}}: Coiling Temperature [°C]
- \text{e}_{\text{tot}}: Total Hot Rolling Conventional Strain [%]
- \text{T}_{\text{fin}}: Finishing Temperature [°C]
- \text{N}_{\text{sol}}: Solubilized (Free) Nitrogen [%]
- \text{Pearl}: Pearlite Fraction Present in Microstructure [%]
- \text{S}_0: Pearlite Lamellar Spacing [mm]

\[ C_{eq} = C + \frac{\text{Mn}}{6} + \frac{\text{Si}}{24} - S \]

\[ \alpha = \frac{T_{\text{fin}} - T_{\text{coil}}}{T_{\text{fin}}} \]

Observations:
- These equations are valid under the following conditions:
  - \text{Slab Reheating Temperature}: 1250°C; \text{T}_{\text{fin}}: 850~880°C; \text{T}_{\text{coil}}: 615~650°C;
  - \text{Final Thickness}: 1.8~4.0 mm; \text{C}: 0.08~0.18%; \text{Mn}: 0.40~1.00%; \text{P} < 0.020%; \text{S} < 0.020%; \text{Si} < 0.030%; \text{Al}: 0.020~0.050%; \text{N}: 0.0030~0.0090%.
C-Mn Mild Steel: Hot/Cold Rolled and Annealed

\[ d_{CR} = 0.013 + 0.28 d_{HR} \] (after 60% cold rolling and annealing)

\[ d_{CR} = 0.011 + 0.29 d_{HR} \] (after 70% cold rolling and annealing)

\[ YS = 3.37 + \frac{2.72}{\sqrt{d_{CR}}} \] \[ YS = 28.16 - 154 d_{HR} \]

\[ \varepsilon_{yield} = 6.27 - 78.5 d_{CR} \]

\[ n = 0.33 - \frac{0.01}{\sqrt{d_{CR}}} \]

Notation:
- \( d_{CR} \): Grain Size of Cold Rolled Strip [mm]
- \( d_{HR} \): Grain Size of Hot Rolled Strip [mm]
- \( YS \): Yield Strength at 0.2% Real Strain [MPa]
- \( \varepsilon_{yield} \): Yield Elongation [%]
- \( n \): Strain Hardening Coefficient Measured during Tension Test

Observations:
- These equations are valid under the following conditions: \( C: 0.005\sim0.10\%; \) \( Mn: 0.40\%; \) \( P < 0.016\%; \) \( S < 0.026\%; \) \( Si < 0.010\%; \) \( Al: < 0.040\%; \) \( N: 0.0020\sim0.0040\%. \)
Cold rolled steel was box annealed at 700°C; the time of treatment, including heating of the samples, was equal to 32 hours, being followed by furnace cooling.


. C-Mn Mild Steel, Full Annealed

\[
n = \frac{5}{10 + \frac{1}{\sqrt{d}}}
\]

Notation:
- \( n \): Strain Hardening Coefficient Measured during Tension Test
- \( d \): Grain Size [mm]


. C-Mn Steels with Ferrite-Pearlite Structure (Grozier & Bucher)

\[
YS = 95.84 + 40.68 \text{Mn} + 70.40 \text{Si} + 1.517 \text{Pearl} + \frac{3.282}{\sqrt{d}}
\]

\[
TS = 223.11 + 56.74 \text{Mn} + 101.97 \text{Si} + 4.323 \text{Pearl} + \frac{2.344}{\sqrt{d}}
\]

Notation:
YS: Yield Strength [MPa]
TS: Tensile Strength [MPa]
Pearl: Pearlite Fraction in Microstructure [%]
Alloy Content: [weight %]
d: Grain Size [mm]

Observations:
- These equations were fitted using at least 50 points of data.
- Useful range: Mn: 0.00 ~ 1.60%; Si: 0.00 ~ 0.80%; Pearl: 0 ~ 80%; d: 0.000252 ~ 0.002770 cm.
- 95% confidence limits: yield strength, ±26 MPa; tensile strength, ±52 MPa.
- Eventually pearlite fraction can be calculated with the equation below:

\[
Pearl = 10.7 + 110.9 \, C + 11.3 \, Mn + 48.4 \, Si
\]

which was fitted used 32 points of date of ferritic-pearlitic hot rolled, air cooled and normalized steel, cooled in air with a mean cooling rate of 1°C/s at 760°C. Its useful range is 0.00~0.30% C; 0.00~1.80% Mn, 0.00~0.25% Si and 0~40% Pearl. Its 95% confidence limit is ±7%; correlation coefficient r is equal to 0.89.

\[ \frac{d\sigma}{d\varepsilon} = 385 + 1.39 \text{Pearl} + 111 \text{Si} + 462 \text{P} + 152 \text{Sn} + 1369 \frac{N_{\text{sol}}}{\sqrt{d}} + 15.4 \]

\[ \varepsilon_{\text{unif}} = 0.27 - 0.016 \text{Pearl} - 0.015 \text{Mn} - 0.040 \text{Si} - 0.043 \text{Sn} - 1.0 \frac{N_{\text{sol}}}{\sqrt{d}} \]

\[ \varepsilon_{\text{tot}} = 1.30 - 0.020 \text{Pearl} + 0.30 \text{Mn} + 0.20 \text{Si} - 3.4 \text{S} - 4.4 \text{P} + 0.29 \text{Sn} + \frac{0.015}{\sqrt{d}} \]

\[ T_{\text{trans}} = 43 + 1.5 \text{Pearl} - 37 \text{Mn} - \frac{6.2}{\sqrt{d}} \]

Notation:
- **YS**: Yield Strength at 0.2% Real Strain [MPa]
- **TS**: Tensile Strength [MPa]
- \( \frac{d\sigma}{d\varepsilon} \): Strain Hardening Coefficient at 0.2% Real Strain [1/MPa]
- **\varepsilon_{\text{unif}}**: Uniform Elongation, Expressed as Real (Logarithmic) Strain
- **\varepsilon_{\text{tot}}**: Total Elongation, Expressed as Real (Logarithmic) Strain
- **Pearl**: Pearlite Fraction in Microstructure [%]
- **\( T_{\text{trans}} \)**: Fracture Appearance Transition Temperature [°C]
- **Alloy Content**: [weight %]
- \( d \): Grain Size [mm]


* Dual Phase Steels
\[ YS = 203 + 855 \sqrt{\frac{1}{L_{aa}}} \]

\[ TS = 266 + 548 \sqrt{\frac{1}{L_{aa}}} + 1741 \sqrt{\frac{f_\beta}{d_\beta}} \]

\[ \frac{d\sigma}{d\varepsilon} = 266 + 548 \sqrt{\frac{1}{L_{aa}}} + 1741 \sqrt{\frac{f_\beta}{d_\beta}} \]

\[ \varepsilon_{unif} = 32 - 64 \sqrt{\frac{1}{L_{aa}}} \]

Notation:
- **LE**: Yield Strength [MPa]
- **LR**: Tensile Strength [MPa]
- **\( \frac{d\sigma}{d\varepsilon} \)**: Strain Hardening Coefficient at Uniform Elongation [1/MPa]
- **\( \varepsilon_{unif} \)**: Uniform Elongation [%]
- **\( L_{aa} \)**: Mean Ferritic Free Path [\( \mu \)m]
- **\( d_\beta \)**: Mean Diameter of Martensite Islands [\( \mu \)m]

Sources:

. Medium C Steels

\[ YS = \sqrt{f_a \left( 35 + 58 \, Mn + \frac{17.4}{\sqrt{d}} \right) + \left( 1 - \sqrt{f_a} \right) \left( 178 + \frac{3.8}{\sqrt{S_0}} \right) + 63 \, Si + 42 \sqrt{N_{sol}} } \]

\[ TS = \sqrt{f_a \left( 246 + 1140 \sqrt{N_{sol}} + \frac{18.2}{\sqrt{d}} \right) + \left( 1 - \sqrt{f_a} \right) \left( 720 + \frac{3.5}{\sqrt{S_0}} \right) + 97 \, Si } \]

\[ ITT = f_a \left( -46 - \frac{11.5}{\sqrt{d}} \right) + \left( 1 - f_a \right) \left[ -335 + \frac{5.6}{\sqrt{S_0}} - \frac{13.3}{\sqrt{p}} + 3.48 \times 10^6 \, t \right] + 48.7 \, Si + 762 \sqrt{N_{sol}} \]

Notation:
- **YS**: Yield Strength at 0.2% Real Strain [MPa]
- **TS**: Tensile Strength [MPa]
- **ITT**: Impact Transition Temperature for 50% Tough Fracture [°C]
- **f**: Volume Fraction of Ferrite
- **d**: Ferrite Grain Size [mm]
- **Alloy Content**: [weight %]
- **N_{sol}**: Solubilized (Free) Nitrogen [%]
- **So**: Pearlite Lamellar Spacing [mm]
- **p**: Pearlite Colony Size [mm]
- **t**: Pearlitic Carbide Lamellar Thickness [mm]

Sources:


**Microalloyed Steels (Hodgson)**

\[
YS = 62.6 + 26.1 \text{ Mn} + 60.2 \text{ Si} + 759.0 \text{ P} + 212.9 \text{ Cu} + 3286.0 \text{ N}_{\text{sol}} + \frac{19.7}{\sqrt{d}} + \Delta \sigma_{\text{ppt}}
\]

\[
TS = 164.9 + 634.7 \text{ C} + 53.6 \text{ Mn} + 99.7 \text{ Si} + 651.9 \text{ P} + 472.6 \text{ Ni} + 3339.4 \text{ N}_{\text{sol}} + \frac{11.0}{\sqrt{d}} + \Delta \sigma_{\text{ppt}}
\]

\[
\Delta \sigma_{\text{ppt Nb}} = 2500 \text{ Nb}
\]

\[
\Delta \sigma_{\text{ppt V}} = 57 \log CR + 700 V + 7800 \text{ N}_{\text{sol}} + 19
\]

**Notation:**
- **YS**: Yield Strength at 0.2% Real Strain [MPa]
- **TS**: Tensile Strength [MPa]
- **Alloy Content**: [weight %]
- **d**: Grain Size [mm]
- **\(\Delta \sigma_{\text{ppt Nb}}\)**: Precipitation Strengthening [MPa], only for steels with Nb [MPa]
- **\(\Delta \sigma_{\text{ppt V}}\)**: Precipitation Strengthening [MPa], only for steels with V [MPa]
- **CR**: Cooling Rate ['C/s]

. Microalloyed Steels (Pickering)

\[
YS = \sigma_0 + 37\text{ Mn} + 83\text{ Si} + 2918 N_{sol} + \frac{15.1}{\sqrt{d}} + \Delta\sigma_{ppt}
\]

Notation:
- \( YS \): Yield Strength at 0.2% Real Strain [MPa]
- \( \sigma_0 \): Friction Stress [MPa]
- \textbf{Alloy Content}: [weight %]
- \( d \): Grain Size [mm]
- \( \Delta\sigma_{ppt} \): Precipitation Strengthening [MPa], for steels with Nb, Ti and/or V, defined by the formula below [MPa].

Observations:
- The Friction Stress \( \sigma_0 \) value depends on the previous treatment of the material and can be found in the table below:

<table>
<thead>
<tr>
<th>Condition</th>
<th>( \sigma_0 ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>70</td>
</tr>
<tr>
<td>Air Cooled</td>
<td>88</td>
</tr>
<tr>
<td>Overaged</td>
<td>62</td>
</tr>
</tbody>
</table>

- The effect of solid solution strengthening from another alloy elements dissolved in ferrite can be included in this equation, using the following linear coefficients:

<table>
<thead>
<tr>
<th>Element</th>
<th>MPa/weight %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni</td>
<td>33</td>
</tr>
<tr>
<td>Cr</td>
<td>-30</td>
</tr>
</tbody>
</table>
The precipitation strengthening contribution is calculated according to the Ashby-Orowan model.

\[
\Delta \sigma_{\text{ppt}} = 5.9 \sqrt{f} \ln \left( \frac{\bar{x}}{2.5 \times 10^{-4}} \right)
\]

Notation:
- **\( \Delta \sigma_{\text{ppt}} \)**: Precipitation Strengthening According to the Ashby-Orowan Model [MPa]
- **\( f \)**: Volume Fraction of the Precipitate
- **\( x \)**: Mean Planar Intercept Diameter of the Precipitate [\( \mu m \)]

Observations:
- Relationship adequate for the calculation of the precipitation strengthening of quench-aged carbides and precipitate carbonitrides in Nb, V and Ti steels.
- \( \Delta \sigma_{\text{ppt}} \) can be calculated using a more simplified approach, multiplying the total content of the precipitating alloy by the factor **B** shown in the table below:

<table>
<thead>
<tr>
<th>Alloy and Precipitate</th>
<th>( B_{\text{min}} ) [MPa/weight %]</th>
<th>( B_{\text{max}} ) [MPa/weight %]</th>
<th>Alloy Range [weight %]</th>
</tr>
</thead>
<tbody>
<tr>
<td>V as V₄C₃</td>
<td>500</td>
<td>1000</td>
<td>0,00 ~ 0,15</td>
</tr>
<tr>
<td>V as VN</td>
<td>1500</td>
<td>3000</td>
<td>0,00 ~ 0,06</td>
</tr>
<tr>
<td>Nb as Nb(CN)</td>
<td>1500</td>
<td>3000</td>
<td>0,00 ~ 0,05</td>
</tr>
<tr>
<td>Ti as TiC</td>
<td>1500</td>
<td>3000</td>
<td>0,03 ~ 0,18</td>
</tr>
</tbody>
</table>
Microalloyed VTiN Steels Processed by Recrystallization Controlled Rolling

\[
YS = 41.4 + 575.20 \frac{C_{eq} + (27401 N_{eq} - 2) \sqrt{V}}{\sqrt{h_f}} + \frac{419.5}{\sqrt{h_f}}
\]

\[
TS = 74.1 + 985.1 \frac{C_{eq} + (31125 N_{eq} - 39) \sqrt{V}}{\sqrt{h_f}} + \frac{181.5}{\sqrt{h_f}}
\]

Notation:
- **YS**: Yield Strength at 0.2% Real Strain [MPa]
- **TS**: Tensile Strength [MPa]
- **Alloy Content**: [weight %]
- **h**: Plate Thickness [mm]

\[
C_{eq} = C + \frac{Mn}{6} + \frac{Cr + Mo}{5} + \frac{Ni + Cu}{15}
\]

\[
N_{eq} = N_{tot} - \frac{Ti}{3.42}
\]

Observations:
- Formula Derived for Steels with Al Content over 0.010% and Si Content between 0.25 and 0.35%.
- Precision of the Formulas: ± 40 MPa.

. Structural Steels: Ductile-Brittle Transition Temperature (Hannula 2015)

\[
DBTT = 42 - \frac{1038}{\sqrt{d_{90}}} + \left(0.341 - \frac{0.047}{\sqrt{d_{90}}}\right)YS
\]

Notation:
- **DBTT 34 J/cm²**: Ductile-Brittle Transition Temperature for Charpy Specific Energy of 34 J/cm² [°C]
- \(d_{90}\): 90th Percentile of Grain Size (Equivalent Circle Diameter) Distribution [μm]
- **YS**: 0.2% Proof Stress [MPa]

Observations:
- Useful range: **DBTT 34 J/cm²**, -50~120°C; \(d_{90}\), 7~26 μm; **YS**, 910~1070 MPa.


. Structural Steels: Impact Transition Temperature (Mintz 1979)

\[
ITT_{27J} = 173 \sqrt{t} - \frac{8.3}{\sqrt{d}} + 0.37 \Delta \sigma_{ppt}
\]

\[
ITT_{54J} = 192 \sqrt{t} - \frac{10.1}{\sqrt{d}} + \frac{\Delta \sigma_{ppt}}{2}
\]
\[ 50\% \text{ FATT} = 131 \sqrt{t} - \frac{12.7}{\sqrt{d}} + 0.45 \Delta \sigma_{ppt} + 46 \]

Notation:
- **ITT 27 J**: Impact Transition Temperature for Charpy Energy of 27 J [°C]
- **ITT 54 J**: Impact Transition Temperature for Charpy Energy of 54 J [°C]
- **50% FATT**: 50% Fibrous Fracture Appearance Transition Temperature [°C]
- \( t \): Carbide Thickness as Measured by Scanning Electron Microscopy [μm]
- \( d \): Grain Size [μm]
- \( \Delta \sigma_{ppt} \): Precipitation Hardening [MPa]

Observations:
- Useful range: \( \text{C}, 0.11\sim0.20\%; \text{Mn}, 0.63\sim1.56\%; \text{Si}, 0.02\sim0.49\%; \text{N}_{\text{total}}, 0.003\sim0.021\%; \text{Nb}_{\text{max}}, 0.071\%; \text{V}_{\text{max}}, 0.20\%; \text{Ti}_{\text{max}}, 0.16\%; \text{Al}_{\text{sol}}, \text{up to} 0.12\%; t, 0.16\sim0.72 \mu m; d, 4.9\sim38.5 \mu m; \Delta \sigma_{ppt}, \text{up to} 225 \text{ MPa.} \)


**Structural Steels: Impact Transition Temperature (Mintz 1994)**

\[
\text{ITT 27J} = 80.1 - \frac{7.41}{\sqrt{d}} + 1.4 \ p - 57.2 \ Si + 1224 \ S - 1360 \ P - 3.7 \sqrt{CR} - 57.8 \ Mn
\]

\[
\text{ITT 54J} = 84.8 - \frac{5.65}{\sqrt{d}} + 1.67 \ p - 53.1 \ Si + 1490 \ S - 1379 \ P - 4.97 \sqrt{CR} - 70.1 \ Mn
\]

Notation:
- **ITT 27 J**: Impact Transition Temperature for Charpy Energy of 27 J [°C] \( - r^2 = 0.950; s = 8.08 \)
**ITT 54 J**: Impact Transition Temperature for Charpy Energy of 54 J [°C] – \( r^2 = 0.955; s = 7.80 \)

- \( d \): Grain Size [μm]
- \( p \): Pearlite Fraction [%]
- **CR**: Cooling Rate [°C/min]

Observations:
- Useful range: \( C, 0.067\text{-}0.220\% ; \ Mn, 0.56\text{-}1.51\% ; \ Si, 0.02\text{-}0.43\% ; \ N_{\text{total}}, 0.0025\text{-}0.010\% ; \ Nb_{\text{max}}, 0.030\% ; \ Al_{\text{max}}, 0.041\% . \)
- Recommended only for normalised steels with ferrite-pearlite microstructure.


**Non-Oriented Si Electrical Steels**

\[
YS = 34.3 + \frac{22.0}{\sqrt{d}} + 258P + 34.2Mn + 52.8Si
\]

\[
TS = 183 + \frac{11.2}{\sqrt{d}} + 506P + 48.7Mn + 109Si + 48.8Al + 2450B
\]

\[
YR = 0.424 + \frac{0.0412}{\sqrt{d}} - 0.078Si - 0.170Al
\]

Notation:
- **YS**: Lower Yield Strength [MPa]
- **TS**: Tensile Strength [MPa]
- **YR**: Yield Ratio
- **d**: Ferrite Grain Size [mm]
**Alloy Content:** [weight %]

**Observations:**
- These equations are valid under the following conditions: **ULC Steel**: Mn: 0.075~0.578%; P < 0.109%; S: 0.003~0.004%; Si < 0.34%; Al: < 0.432%; N: 0.0014~0.0020%; B < 0.0030%.
- Cold rolled steel was box annealed at 700°C; the time of treatment, including heating of the samples, was equal to 32 hours, being followed by furnace cooling.


\[ P_r = 0.658 - 0.474 Si - 2.311 Al - 25.99 O + 12.51 C + 123.7 S_{oel} + 130.2 \Delta S - 137.5 N + 5.266 h \]

**Notation:**
- \( P_r \): Core Loss [W/kg]
- **Alloy Content**: [weight %]
- \( h \): Thickness [mm]

**Observations:**
- This equation is valid under the following conditions: C: 0.002~0.040%; S: 0.004~0.015%; N: 0.003~0.007%.
- Negative effects of O and N are in direct contradiction with specific experimental results.
- Adjusted Squared Multiple Correlation: 0.823; Residual Mean Square: 0.082 W²/kg²

\[ P_r = 4.29 + 66.4 C + 0.0282 GBI + 16.2 \frac{h^2}{\rho r} \]

**Notation:**
- \( P_r \): Total Core Loss at 15 KG [W/kg]
**Alloy Content**: [weight %]

**GBI**: Number of Grain Boundary Intercepts per mm

**h**: Thickness [mm]

**ρ**: Density [g/mm³]

**r**: Resistivity [μΩ.mm]

**Schaeffler Diagram**

Source: Air Products Web Site
(http://www.airproducts.com/maxx/software/UK/WeldingFaultFinder/wff22413.html)
- **Shear Modulus of Steel and its Phases**

  . **Ferrite**

  \[
  \mu = 64000 \left[ 1 - \frac{(T - 300)}{2235} \right] \quad (-273°C < T < 300°C)
  \]

  \[
  \mu = 64000 \left[ 1 - \frac{(T - 300)}{2235} \right] - 0.032 (T - 573)^2 \quad (300°C \leq T < 700°C)
  \]

  \[
  \mu = 64000 \left[ 1 - \frac{(T - 300)}{2235} \right] - 0.032 (T - 573)^2 - 0.024 (T - 923)^2 \quad (700°C \leq T < 770°C)
  \]

  \[
  \mu = 69200 \left[ 1 - \frac{(T - 300)}{1382} \right] \quad (770°C \leq T < 911°C)
  \]

  . **Austenite**

  \[
  \mu = 81000 \left[ 1 - \frac{(T - 300)}{1989} \right] \quad (911°C \leq T < 1392°C)
  \]

  . **Delta Ferrite**

  \[
  \mu = 39000 \left[ 1 - \frac{(T - 300)}{2514} \right] \quad (1392°C \leq T < 1537°C)
  \]

  **Notation:**
\( \mu \): Shear Modulus [MPa]
\( T \): Temperature [K]

- **Sheet and Plate Cutting Force and Work**

  . **Mesquita**

  \[ K_c = 0.88 \cdot TS \]

  \[ F_c = t \cdot P \cdot K_c \]

  Notation:
  - \( K_c \): Cutting Specific Pressure or Shear Stress [MPa]
  - \( TS \): Tensile Strength [MPa]
  - \( F_c \): Cutting Force [N]
  - \( t \): Thickness [mm]
  - \( P \): Cutting Perimeter [mm]


  . **Tschaetsch**

  \[ F = W \cdot t \cdot \tau_b \]

  \[ W = \frac{a \cdot F \cdot t}{1000} \]

  \[ P = \frac{F \cdot v}{\eta} \]

  Notation:
  - \( F \): Cutting Force [N]
**W**: Width [mm]  
**t**: Thickness [mm]  
**W**: Cutting Work [N.mm]  
**a**: Mean Force/Maximum Force Ratio (≈ 0.6 for shearing)  
**P**: Cutting Power [W]  
**v**: Shear Speed [m/s]  
**η**: Machine Efficiency (≈ 0.7)  
**τ_B**: Shear Stress [MPa], as defined by the table or formulas described in the observation below.

**Observations:**
- The value of **τ_B** can be calculated from the following equations, where **C** is the carbon weight content of steel:
  - Hot rolled or annealed steel (soft) – \( r^2 = 0.992 \), Standard Error of Deviation = 13 MPa:
    \[
    \tau_B = 223 + 550C
    \]
  - Cold rolled steel (hard) – \( r^2 = 0.988 \), Standard Error of Deviation = 9 MPa:
    \[
    \tau_B = 249 + 786C
    \]
- These equations were fitted using the **τ_B** data available below, expressed in N/mm²:

<table>
<thead>
<tr>
<th>Steel</th>
<th>C</th>
<th>Mn</th>
<th>Si</th>
<th>Soft</th>
<th>Hard</th>
</tr>
</thead>
<tbody>
<tr>
<td>St 12</td>
<td>0.10 max</td>
<td>0.50 max</td>
<td>-</td>
<td>240</td>
<td>300</td>
</tr>
<tr>
<td>St 13</td>
<td>0.10 max</td>
<td>0.50 max</td>
<td>-</td>
<td>240</td>
<td>300</td>
</tr>
<tr>
<td>St 14</td>
<td>0.08 max</td>
<td>0.40 max</td>
<td>-</td>
<td>250</td>
<td>320</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>St 37</td>
<td>0.20 max</td>
<td>1.25 max</td>
<td>0.25 max</td>
<td>310</td>
<td>-</td>
</tr>
<tr>
<td>St 42</td>
<td>0.25 max</td>
<td>1.25 max</td>
<td>0.25 max</td>
<td>400</td>
<td>-</td>
</tr>
<tr>
<td>C10</td>
<td>0.08-0.13</td>
<td>0.30-0.60</td>
<td>0.30 max</td>
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<tr>
<td>C20</td>
<td>0.18-0.23</td>
<td>0.30-0.70</td>
<td>0.30 max</td>
<td>320</td>
<td>380</td>
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<tr>
<td>C30</td>
<td>0.27-0.34</td>
<td>0.50-0.80</td>
<td>0.10-0.40</td>
<td>400</td>
<td>500</td>
</tr>
<tr>
<td>C60</td>
<td>0.57-0.65</td>
<td>0.60-0.90</td>
<td>0.15-0.35</td>
<td>550</td>
<td>720</td>
</tr>
</tbody>
</table>

- Solidus Temperature of Steels

. Qian

\[ T_{\text{Sol}} = a - b \, C_e \]

\[ C_e = \frac{80.5 \, C + 33.5 \, (S + P) + 3.75 \, Mn + 17.8 \, Si + 3.4 \, Co + 3.8 \, Al + 1.5 \, Cr + 3 \, Ni}{80.5} \]

Notation:
- \( T_{\text{Sol}} \): Steel Solidus Temperature [°C]
- \( a \): Constant equal to
  - 1493, for 0.1 ≤ \( C_e \) ≤ 0.2%;
  - 1534, for \( C_e < 0.1\% \) or \( C_e > 0.2\% \);
- \( b \): Constant equal to
  - 0, for 0.1 ≤ \( C_e \) ≤ 0.2%;
  - 410, for \( C_e < 0.1\% \);
  - 184, for \( C_e > 0.2\% \);
- **Alloy Content**: [weight %]


. Takeuchi

\[ T_{\text{Sol}} = 1536 - \left[ 415.5 \, C + 12.3 \, Si + 6.8 \, Mn + 124.5 \, P + 183.9 \, S + 4.3 \, Ni + 1.4 \, Cr + 4.1 \, Al \right] \]

Notation:
- \( T_{\text{Sol}} \): Steel Solidus Temperature [°C]
- **Alloy Content**: [weight %]
- **Specimen Orientation for Mechanical Testing**

![Diagram of Specimen Orientation for Mechanical Testing](image)
Notes:

- The first letter denotes the direction of the applied main tensile stress.

- The second letter denotes the direction of crack propagation.

Steel Properties Map

Source: [www.tecmetal.com.br](http://www.tecmetal.com.br)
- Thermal Properties of Steel

. BISRA

<table>
<thead>
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<th>T</th>
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<th>1040</th>
<th>1524</th>
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</table>
Notation:
- $k$: Thermal Conductivity [W/(m.°C)]
- $c$: Specific Heat Capacity [J/(kg.ºC)]
- $H$: Enthalpy [J/kg]
- $T$: Temperature [ºC]

Observations:
- Chemical composition of the steels [wt %]:

<table>
<thead>
<tr>
<th>Steel</th>
<th>C</th>
<th>Mn</th>
<th>Si</th>
<th>P</th>
<th>S</th>
<th>Cu</th>
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<td>0.11</td>
<td>0.031</td>
<td>0.029</td>
<td>0.12</td>
</tr>
</tbody>
</table>

### Chou

\[
k = 80.91 - 9.9269 \times 10^{-2} T + 4.613 \times 10^{-5} T^2 \quad (T \leq \text{Ar}_3)
\]

\[
k = 20.14 + 9.313 \times 10^{-3} T \quad (T > \text{Ar}_3)
\]

\[
c = -4720.324 + 4.583364T + \frac{1.109483 \times 10^9}{T^2} \quad (800K < T < 1000K)
\]

\[
c = -11501.07 + 12.476362T \quad (1000K < T < 1042K)
\]

\[
c = 34871.21 + 32.026587T \quad (1042K < T < 1060K)
\]

\[
c = -10068.18 + 5.98686T + \frac{5.217657 \times 10^9}{T^2} \quad (1060K < T < 1184K)
\]

\[
c = 429.8495 + 0.1497802T \quad (1084K < T < 1665K)
\]

**Notation:**
- \(k\): Thermal Conductivity [J/m.K.s]
- \(c\): Specific Heat Capacity [J/kg.K]
- \(T\): Temperature [K]

**. Krzyzanowski**

\[ c_p = 422.7 + 48.66 \exp(0.319 \times 10^{-5} \, T) \]  \hspace{1cm} (T \leq 700°C)

\[ c_p = 657.0 + 0.084 \left( \frac{T}{1000} \right)^{-24.6} \]  \hspace{1cm} (T > 700°C)

\[ \lambda = 23.16 + 51.96 \exp(-2.02519 \times 10^{-3} \, T) \]

\[ \rho = \frac{7850}{\left(1 + 0.004 \times 10^{-6} \, T^2\right)^3} \]

Notation:
- \( c_p \): Specific Heat [J/kg.°C]
- \( T \): Temperature [°C]
- \( \lambda \): Thermal Conductivity [J/m.°C.s]
- \( \rho \): Density [kg/m³]


**. Seredynski**

\[ k = -58.6 \times 10^{-3} \, T + 72.5 \]  \hspace{1cm} (T < 810°C)
\[ k = 10.75 \times 10^{-3} T + 16.8 \quad (T > 810^\circ C) \]
\[ D = 0.15 \times 10^{-7} T - 0.07825 \times 10^{-4} \quad (700^\circ C < T < 875^\circ C) \]
\[ D = 0.02667 \times 10^{-7} T - 0.02966 \times 10^{-4} \quad (T > 875^\circ C) \]
\[ \varepsilon = \frac{T}{1000} \left( 0.12491 - \frac{T}{1000} - 0.38012 \right) + 1.0948 \]

Notation:
- \( k \): Thermal Conductivity [J/m.°C.s]
- \( D \): Thermal Diffusivity [m²/s]
- \( \varepsilon \): Emissivity
- \( T \): Temperature [°C]

Observation:
- Formulas specific for BS En 3 or SAE 1021 steel: 0.17-0.23% C; 0.60-0.90% Mn


. Touloukian

\[ \varepsilon = \frac{0.85}{1 + \exp(42.68 - 0.02682T_{sup})^{0.0115}} \]

Notation:
- \( \varepsilon \): Emissivity
\( T_{\text{sup}} \): Superficial Temperature [K]

- **Thermal Properties of Steel Scale**

  **Krzyzanowski**

\[ k = 1 + 7.833 \times 10^{-4} T \quad (873 K < T < 1473 K) \]

\[ c = 674.969 + 0.297 T - 4.367 \times 10^{-5} T \quad (600°C < T < 1100°C) \]

\[ E = 240 \left[ 1 - 4.7 \times 10^{-4} (T - 25) \right] \]

Notation:
- **k**: Conductivity [W/m.K]
- **c**: Specific Heat Capacity [J/kg.°C]
- **E**: Young’s Modulus [GPa]
- **T**: Temperature [°C]

### Thermomechanical Processing of Steel

<table>
<thead>
<tr>
<th>Structure</th>
<th>Temperature</th>
<th>Type of Processing</th>
<th>Conventional Processes</th>
<th>Thermo-Mechanical Processes</th>
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<tbody>
<tr>
<td>Recrystallized Aустенит</td>
<td>Normal Slab Heating Temp</td>
<td>AR, N, CR(NR), QT, TM</td>
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<td></td>
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<tr>
<td>Recrystallized Aустенит</td>
<td>Normalizing of Quenching Temp</td>
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<td>Non-recrystallized Aустенит</td>
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<td>Aустенит + Феррит</td>
<td>Arc of Ac</td>
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<td></td>
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<tr>
<td>Феррит + Феррит or Феррит + Бейнит</td>
<td>Tempering Temp</td>
<td></td>
<td></td>
<td></td>
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</table>

**Notes:**
- **AR:** As Rolled
- **N:** Normalizing
- **CR(NR):** Controlled Rolling (Normalizing Rolling)
- **QT:** Quenching and Tempering
- **TM:** Thermo-Mechanical Rolling (Thermo-Mechanical Controlled Process)
- **R:** Reduction
- **(·):** Sometimes rolling in the dual-phase temperature region of austenite and ferrite
- **Acc:** Accelerated Cooling

**Source:** *Requirements Concerning Materials and Welding. IACS – International Association of Classification Societies Requirement 1975*, Revision 2, 2004, 228 p.
- **Time-Temperature Equivalency Parameters for Heat Treating**

. **Austenitizing (Anisothermal)**

In this case the Austenitization Time-Temperature Equivalence Parameter in Terms of Grain Size, $P_a$, is the period of heating/cooling time between $T_{\text{max}}$ and $T_{\text{min}}$, where

$T_{\text{max}}$: Maximum Temperature during the Austenitizing Treatment [$^\circ\text{C}$];

$T_{\text{min}}$: Temperature Calculated According to the Following Equation [$^\circ\text{C}$]:

$$T_{\text{min}} = T_{\text{max}} - \frac{R T_{\text{max}}^2}{\Delta H_a}$$

Notation:

$R$: Molar Gas Constant, 8.314 JK$^{-1}$mol$^{-1}$

$\Delta H_a$: Activation Energy of Austenitic Grain Coarsening, 460 kJmol$^{-1}$ for low alloy steels


. **Austenitizing (Isothermal)**

$$P_a = \frac{1}{\left[\frac{1}{T_a} - \frac{2.3 R}{\Delta H_a \log t_a}\right]}$$

Notation:

$P_a$: Austenitization Time-Temperature Equivalence Parameter in Terms of Grain Size [K]
\[ t_{0.05} = \frac{0.3 l^2}{D_0 e^{Q/R}} \]

Notation:
- \( t_{0.05} \): Treatment Time Necessary to Achieve 5% Microstructure Banding [min]
- \( l \): Mean Spacing between Bands [mm]
- \( D_0 \): Diffusion Constant for the Alloy Element being Considered [cm²/s]:
  - P: 0.01 cm²/s
  - Mn: 0.16 cm²/s
- \( Q \): Activation Energy for the Alloy Element Being Considered [cal/mol]:
  - P: 43700 cal/mol
  - Mn: 62500 cal/mol
- \( R \): Molar Gas Constant, 1.987 JK⁻¹mol⁻¹
- \( T \): Austenitization Temperature [K]

. **Tempering (Anisothermal)**

\[ P = T (c + \log \tau) \]

\[ \tau = t + \frac{T}{2.303 k_1 (c - \log k_1)} + \frac{T}{2.303 k_2 (c - \log k_2)} \]

**Notation:**
- \( P \): Hollomon-Jaffe Parameter [K]
- \( T \): Tempering Temperature [K]
- \( c \): Constant Characteristic of the Steel Being Tempered
- \( t \): Soaking Time Under \( T \) [h]
- \( k_1 \): Heating Rate [K/hour]
- \( k_2 \): Cooling Rate [K/hour]

**Observations:**
- See the section **Tempering (Isothermal)** for the values of constant \( c \).

**Sources:**

. **Tempering (Isothermal)**

\[ P = T (c + \log t) \]

\[ c = 21.53 - 5.8 C \]

**Notation:**
\( P \): Hollomon-Jaffe Parameter [K]
\( T \): Tempering Temperature [K]
\( c \): Constant Characteristic of the Steel Being Tempered
\( t \): Soaking time under \( T \) [h]
\( C \): Carbon Content [wt%]

Observations:
- Other values for the constant \( c \) were proposed by several authors for carbon, microalloyed and low alloy steels:
  - 18 (Grange & Baughman)
  - 20 (Larson & Miller, Irvine, Thelning)
- This expression was also deduced by Larson & Miller, which applied it to the study of metal creep. In that case \( c \) is equal to 20 and \( P \) is divided by 1000. Such relationship was also used for the study of hydrogen resistance and HAZ hardness of steels.

Sources:
- **Welding Effects**

  . **Weld Interface Cracking Susceptibility during Flash Butt Welding**

  \[
  F_{eq} = (C - 0.03) \left[ Si^2 + \left( \frac{Mn}{10} \right)^2 + (4 Al)^2 + \left( \frac{3 Cr}{2} \right)^2 \right]
  \]

  Notation:
  - \(F_{eq}\): Weld Interface Cracking Susceptibility during Flash Butt Welding (No Crack = Zero)
  - **Alloy Content**: [weight %]


  . **Tensile Strength after Flash Butt Welding**

  \[
  TS_{eq} = 52 \left( C + \frac{Mn}{5} + \frac{Si}{7} + \frac{Cr}{9} + \frac{V}{2} \right) + 30
  \]

  Notation:
  - \(TS_{eq}\): Tensile Strength After Flash Butt Welding [kgf/mm²]
  - **Alloy Content**: [weight %]

- **Welding Pool Phenomena**

- **Young Modulus**

  **Definition**

  \[ E = 2G(1 + \nu) \]

  Notation:
  - \( E \): Young Modulus
  - \( G \): Shear Modulus
  - \( \nu \): Poisson Ratio

  . Elastic Range: 0.3
  . Plastic Range: 0.5


  **Steel, High Temperature: Pietrzyk**

  \[
  E = \left[ 2.07 + 0.87438 \left( \frac{T}{1000} \right) - 10.0906 \left( \frac{T}{1000} \right)^2 + 14.48466 \left( \frac{T}{1000} \right)^3 - 6.20767 \left( \frac{T}{1000} \right)^4 \right] 10^5
  \]

  Notation:
  - \( E \): Young Modulus [MPa]
  - \( T \): Temperature [°C]

  Observation:
  - Valid for steel. No information available about the range of valid temperatures.
Steel, High Temperature: Tselikov

\[
E = 308250 + 42924C - 144000C^2 + 20525Si - 5289Mn - 12000P + 174000S - 225.6T + 0.01379T^2
\]

Notation:
- \(E\): Young Modulus [kgf/cm²]
- \(C\): C content [weight %]
- \(Mn\): Mn content [weight %]
- \(Si\): Si content [weight %]
- \(P\): P content [weight %]
- \(S\): S content [weight %]
- \(T\): Temperature [°C]

Observation:
- Valid for carbon, alloy and stainless steels between 20 and 900°C.

APPENDIXES
### Greek Letters

<table>
<thead>
<tr>
<th>Upper Case</th>
<th>Lower Case</th>
<th>Name</th>
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<tr>
<td>Ω</td>
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</tbody>
</table>
- **Statistical Formulas - General**

**Correlation Coefficient**

\[
 r = \pm \sqrt{\frac{\sum (Y_{est} - \bar{Y})^2}{\sum (Y - \bar{Y})^2}}
\]

Notation:
- \( r \): Correlation Coefficient
- \( Y \): Raw Data
- \( Y_{est} \): Estimated Data Calculated by the Fitted Equation
- \( \bar{Y} \): Mean of the Raw Data


**Root Mean Square Deviation**

\[
 \sigma = \sqrt{\frac{\sum (Y_{est} - Y_{real})^2}{n}}
\]

Notation:
- \( \sigma \): Root Mean Square Deviation
- \( Y_{est} \): Estimated Data Calculated by the Fitted Equation
\( Y_{\text{real}} \): Real Data
\( n \): Number of Points of Data

- **Trigonometry**

**Trigonometrical Relations**

![Diagram of trigonometric functions](image)

Versine: \( \text{versin}(\theta) = 1 - \cos(\theta) \)

Vercosine: \( \text{vercosin}(\theta) = 1 + \cos(\theta) \)

Coversine: \( \text{coversin}(\theta) = 1 - \sin(\theta) \)

Covercosine: \( \text{covercosine}(\theta) = 1 + \sin(\theta) \)

Haversine: \( \text{haversin}(\theta) = \text{versin}(\theta)/2 \)

Havercosine: \( \text{havercosin}(\theta) = \text{vercosin}(\theta)/2 \)

Hacoversine: \( \text{hacoversin}(\theta) = \text{coversin}(\theta)/2 \)

Hacovercosine: \( \text{hacovercosine}(\theta) = \text{covercosine}(\theta)/2 \)

Exsecant: \( \text{exsec}(\theta) = \sec(\theta) - 1 \)

Excosecant: \( \text{excsc}(\theta) = \csc(\theta) - 1 \)
## Unit Conversions

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<tr>
<td>F</td>
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<td>°C</td>
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<td>J ou N.m</td>
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<td>lbf</td>
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<tr>
<td>lbf/in²</td>
<td>1</td>
<td>psi</td>
</tr>
<tr>
<td>From</td>
<td>Multiply by</td>
<td>To</td>
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<tr>
<td>------------------</td>
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<td>----------</td>
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<tr>
<td>long ton</td>
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<tr>
<td>oz</td>
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<tr>
<td>Pa</td>
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- **Useful Data and Constants**

### API Pipeline Steels – Mechanical Properties

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<th>Grade</th>
<th>Yield Strength, Minimum (MPa)</th>
<th>Yield Strength, Maximum (MPa) only for PSL 2</th>
<th>Ultimate Tensile Strength, Minimum (MPa)</th>
<th>Ultimate Tensile Strength, Maximum (MPa) only for PSL 2</th>
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<td>496</td>
<td>414</td>
<td>758</td>
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<tr>
<td>X46</td>
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<td>524</td>
<td>434</td>
<td>758</td>
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<td>531</td>
<td>455</td>
<td>758</td>
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<td>X56</td>
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<td>544</td>
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<td>758</td>
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<td>X60</td>
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<td>517</td>
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<td>X85</td>
<td>448</td>
<td>600</td>
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<tr>
<td>X70</td>
<td>483</td>
<td>621</td>
<td>565</td>
<td>758</td>
</tr>
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<td>621</td>
<td>827</td>
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<tr>
<td>X90 (for PSL 2)</td>
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<td>775</td>
<td>695</td>
<td>915</td>
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<tr>
<td>X100 (for PSL 2)</td>
<td>690</td>
<td>840</td>
<td>760</td>
<td>990</td>
</tr>
<tr>
<td>X120 (for PSL 2)</td>
<td>830</td>
<td>1050</td>
<td>915</td>
<td>1145</td>
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**Fuels and Combustion Gases:**

- **Density (Gas)**
  - Natural Gas: 0.81 kg/Nm³
  - Butane: 2.44 kg/Nm³
  - Propane: 1.85 kg/Nm³
  - Liquified Petroleum Gas (LPG): 2.29 kg/Nm³
  - Air: 1.27 kg/Nm³

- **Density (Liquid)**
  - Butane: 0.58 kg/l
  - Propane: 0.51 kg/l
  - Liquified Petroleum Gas (LPG): 0.54 kg/Nm³
  - Water: 1.00 kg/Nm³
Heat Capacity in Function of Temperature

Heat Capacity $[\text{kcal/°C m}^3] = a + bT [\text{°C}]$. Values of $a$ and $b$ for some gases are seen below:

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<tr>
<th>Gas</th>
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<th>$b$</th>
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<tr>
<td>$\text{C}_2\text{H}_6$</td>
<td>0.600</td>
<td>0.000540</td>
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<tr>
<td>$\text{C}_3\text{H}_8$</td>
<td>0.871</td>
<td>0.001226</td>
</tr>
<tr>
<td>$\text{CH}_4$</td>
<td>0.380</td>
<td>0.000210</td>
</tr>
<tr>
<td>$\text{CO}$</td>
<td>0.302</td>
<td>0.000022</td>
</tr>
<tr>
<td>$\text{CO}_2$</td>
<td>0.406</td>
<td>0.000090</td>
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<tr>
<td>$\text{H}_2$</td>
<td>0.301</td>
<td>0.000200</td>
</tr>
<tr>
<td>$\text{N}_2$</td>
<td>0.302</td>
<td>0.000022</td>
</tr>
<tr>
<td>$\text{O}_2$</td>
<td>0.320</td>
<td>0.000059</td>
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</table>

Net Heating Value

- Acetylene ($\text{C}_2\text{H}_2$): 13412 Kcal/Nm$^3$
- Basic Oxygen Steelmaking Off-Gas (OG Gas): 770 kcal/Nm$^3$
- Blast Furnace Gas: 770 Kcal/Nm$^3$
- Benzene ($\text{C}_6\text{H}_6$): 33,823 Kcal/Nm$^3$
- Butane ($\text{C}_4\text{H}_{10}$): 29,560 Kcal/Nm$^3$
- Butene/Buthylene ($\text{C}_4\text{H}_8$): 27,900 Kcal/Nm$^3$
- Charcoal: 6,800 kcal/kg
- Carbon Monoxide (CO): 3,019 Kcal/Nm$^3$
- Coke Oven Gas: 4,400 Kcal/Nm$^3$
- Diesel Oil: 10.200 kcal/kg
- Electricity: 860 kcal/kW
- Ethane ($\text{C}_2\text{H}_6$): 15,236 Kcal/Nm$^3$
- Ethene/Ethylene ($\text{C}_2\text{H}_4$): 14,116 Kcal/Nm$^3$
- Fuel Oil: 8,640–9,000 kcal/l or 9,600 ~ 10,000 kcal/kg
- Hexane ($\text{C}_6\text{H}_{14}$): 41,132 Kcal/Nm$^3$
- Hydrogen (H): 2,582 Kcal/Nm$^3$
. Hydrogen Sulfide (H₂S): 5,527 Kcal/Nm³
. i-Butane (C₄H₁₀): 28,317 Kcal/Nm³
. i-Pentane (C₅H₁₂): 34,794 Kcal/Nm³
. Liquified Petroleum Gas (LPG): 25,300 ~ 27,300 kcal/Nm³
. Methane (CH₄): 8,557 Kcal/Nm³
. Natural Gas: 9,000 ~ 9,400 Kcal/Nm³
. Pentane (C₅H₁₂): 34,943 Kcal/Nm³
. Propane (C₃H₈): 21,809 Kcal/Nm³
. Propene/Propylene (C₃H₆): 20,550 Kcal/Nm³
. Toluene (C₇H₈): 40,182 Kcal/Nm³
. Xylene (C₈H₁₀): 46,733 Kcal/Nm³
. Wood: 2,500 kcal/kg

- **Typical Chemical Compositions**

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<th>N₂</th>
<th>H₂</th>
<th>CH₄</th>
<th>C₂H₆</th>
<th>C₃H₈</th>
<th>CO</th>
<th>CO₂</th>
<th>O₂</th>
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</thead>
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<tr>
<td>Coke Oven Gas</td>
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<td>61.55</td>
<td>24.54</td>
<td>0.42</td>
<td>0.06</td>
<td>8.04</td>
<td>0.00</td>
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<td>Natural Gas</td>
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<td>----</td>
<td>87.91</td>
<td>7.08</td>
<td>1.91</td>
<td>----</td>
<td>0.59</td>
<td>0.00</td>
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</table>

**Mathematical Constants**

- e: 2.718281828
- Pi: 3.141592654

**Physical Constants**

- Acceleration of gravity: g = 9.805 m/s²
- Avogrado: Nₐ = 6.022 x 10⁻²³ 1/mol
- Boltzmann: k = 1.38065 x 10⁻²³ J/K
- Ideal Gas Constant R:
  - 1.98717 cal/(K mol)
  - 82.056 cm³ atm/(K.mol)
  - 0.082056 l atm/(K.mol)
  - 8.31433 x 10⁷ erg/(K.mol)
  - 8.31433 J/(K.mol)
- Stefan-Boltzmann: $\sigma = 5.6704 \times 10^{-8}$ W/(m² K⁴) or J/(s m² K⁴)

**Physical Properties of Scale (Iron Oxide)**

- Thermal Conductivity:
  - Industrial Scale: 3.0 W/(m.K)
  - Hematite (Fe₂O₃): 1.2 W/(m.K)
  - Magnetite (Fe₃O₄): 1.5 W/(m.K)
  - Wustite (FeO): 3.2 W/(m.K)
- Density:
  - Industrial Scale:
    - 4.86 g/cm³ (Combustol)
    - 5.00 g/cm³ (Picqué)
    - 5.70 g/cm³ (Krzyzanowski)
  - Hematite (Fe₂O₃): 4.90 g/cm³
  - Magnetite (Fe₃O₄): 5.00 g/cm³
  - Wustite (FeO): 5.70 g/cm³
  - Bulk, Porous Scale as Raw Material, Room Temperature:
    - 2.40~2.89 t/m³
    - Stowage Factor: 0.38 m³/t
- Hardness:
  - Hematite (Fe₂O₃): 1000 HV
  - Magnetite (Fe₃O₄): 320 ~ 500 HV
  - Wustite (FeO): 270 ~ 350 HV
- Iron Content in Scale: 74.6% (Stoichiometric)
- Linear Coefficient of Thermal Contraction:
  . Fe: $19 \times 10^{-6}$ m/°C
  . Wustite: $14 \times 10^{-6}$ m/°C
- Melting Point:
  . Fayalite ($2\text{FeO}.\text{SiO}_2$): 1177°C
- Specific Heat:
  . Industrial Scale: $766$ J/(kg.K) (600°C)
  . Hematite ($\text{Fe}_2\text{O}_3$): $980$ J/(kg.K)
  . Magnetite ($\text{Fe}_3\text{O}_4$): $870$ J/(kg.K)
  . Wustite ($\text{FeO}$): $725$ J/(kg.K)
- Thermal Expansion Coefficient:
  . Ferrite (0 ~ 900°C): $15.3 \times 10^{-6}$ K⁻¹
  . Hematite ($\text{Fe}_2\text{O}_3$):
    - 20 ~ 900°C: $14.9 \times 10^{-6}$ K⁻¹
    - 100 ~ 300°C: $10.8 \times 10^{-6}$ K⁻¹
    - 100 ~ 1000°C: $12.2 \times 10^{-6}$ K⁻¹
    - 400 ~ 800°C: $13.0 \times 10^{-6}$ K⁻¹
  . Magnetite ($\text{Fe}_3\text{O}_4$): $1.5$ W/(m.K)
    - 25°C: $11.0 \times 10^{-6}$ K⁻¹
    - 400°C: $14.0 \times 10^{-6}$ K⁻¹
    - 550°C: $27.0 \times 10^{-6}$ K⁻¹
  . Wustite ($\text{FeO}$):
    - 100 ~ 1000°C: $12.2 \times 10^{-6}$ K⁻¹
    - 400 ~ 800°C: $15.0 \times 10^{-6}$ K⁻¹

### Physical Properties of Steel and its Microstructural Constituents

- Densities:
  . Bulk Steel: 7850 kg/m³
  . Ferrite (Fe α):
    - Caballero: 7882 kg/m³
- Jablonka: 7870 kg/m³ (20°C)
  - Cementite (Fe₃C):
    - Caballero: 7687 kg/m³
    - Jablonka: 7685 kg/m³ (20°C)
  - NbC: 7790 kg/m³
  - VC: 5700 kg/m³
- Electrical Resistivity at 15.6°C: 17 x 10⁻⁸ Ω.m
- Emissivity of Polished Metal Surface:
  - 0.07 @ 38°C
  - 0.10 @ 260°C
  - 0.14 @ 540°C
- Emissivity of Oxidized Steel Plate at 15.6°C: 0.80
- Heat Transfer Coefficient at Scale/Steel Interface: 30,000 W/m².K
- Lattice Parameters (Ambient Temperature):
  - Ferrite (pure Fe): 2.866 Å
  - Cementite: a = 4.5246 Å, b = 5.0885 Å, c = 6.7423 Å
- Linear Coefficient of Thermal Expansion:
  - Bulk: 11.7 x 10⁻⁶ °C⁻¹
  - Ferrite: 1.244 x 10⁻⁵ °C⁻¹
  - Austenite: 2.065 x 10⁻⁵ °C⁻¹
- Melting Point: 1300 ~ 1450°C
- Modulus:
  - Bulk: 159,000 MPa
  - Shear: 83,000 MPa
  - Young: 207,000 MPa
- Poisson’s Ratio:
  - Elastic Range: 0.3
  - Plastic Range: 0.5
- Specific Heat: 0.12 cal/g.°C
- Speed of Sound through Steel: 5,490 m/s
- Thermal Conductivity at 15.6°C: 58.9 W/m.K
- Volumetric Coefficient of Thermal Expansion: 35.1 x 10⁻⁶ °C⁻¹
Sources:


